

CS 490:
NATURAL LANGUAGE
PROCESSING

Dan Goldwasser, Abulhair Saparov

Lecture 12: Syntax II

PARSING CFGS

- The **CKY** (Cocke-Kasami-Younger; or **CYK**) algorithm is a simple application of dynamic programming to parsing CFGs in Chomsky normal form.
- The algorithm was described by **K**asami (1965) and **Y**ounger (1967), and rediscovered later by **C**ocke and Schwartz (1970).

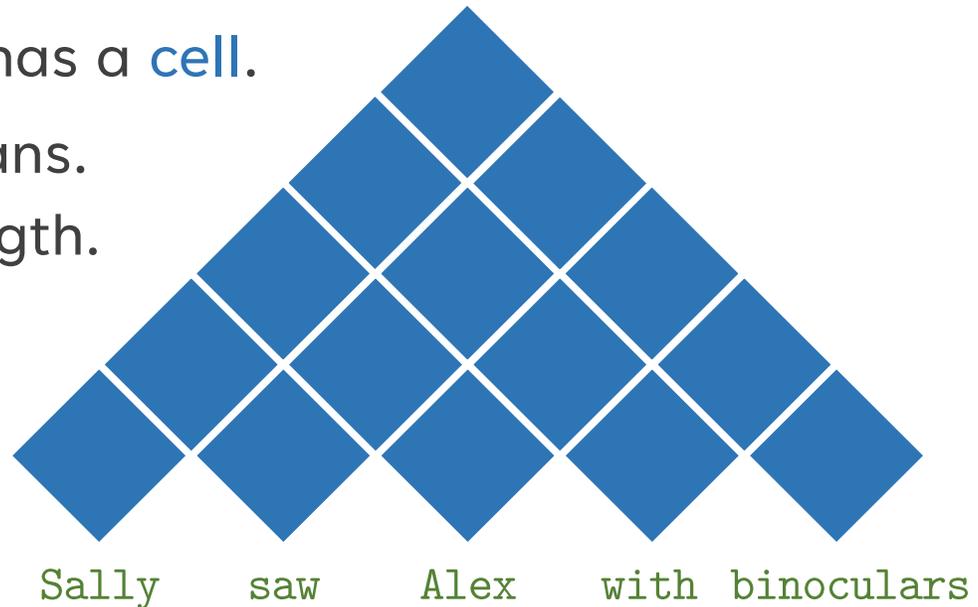
PARSING CFGS

- The **CKY** (Cocke-Kasami-Younger; or **CYK**) algorithm is a simple application of dynamic programming to parsing CFGs in Chomsky normal form.
- In dynamic programming, we solve a problem by breaking it down into simpler subproblems.
 - The solutions to the subproblems makes it easier to solve the full problem.
- In parsing CFGs,
 - Suppose we are given a sentence ‘Sally saw Alex with binoculars.’
 - And suppose we know that ‘Sally’ is a noun phrase (**NP**) and ‘saw Alex with binoculars’ is a verb phrase (**VP**).
 - Our grammar contains the rule **S** → **NP VP**.
 - We can conclude that the full sentence can be parsed as **S**.

CKY PARSING

- The **CKY** (Cocke-Kasami-Younger; or **CYK**) algorithm is a simple application of dynamic programming to parsing CFGs in Chomsky normal form.
- We use a data structure called a **chart**.
 - For each span (i, j) the chart has a **cell**.
- Start parsing with the smallest spans.
 - Progressively increase span length.

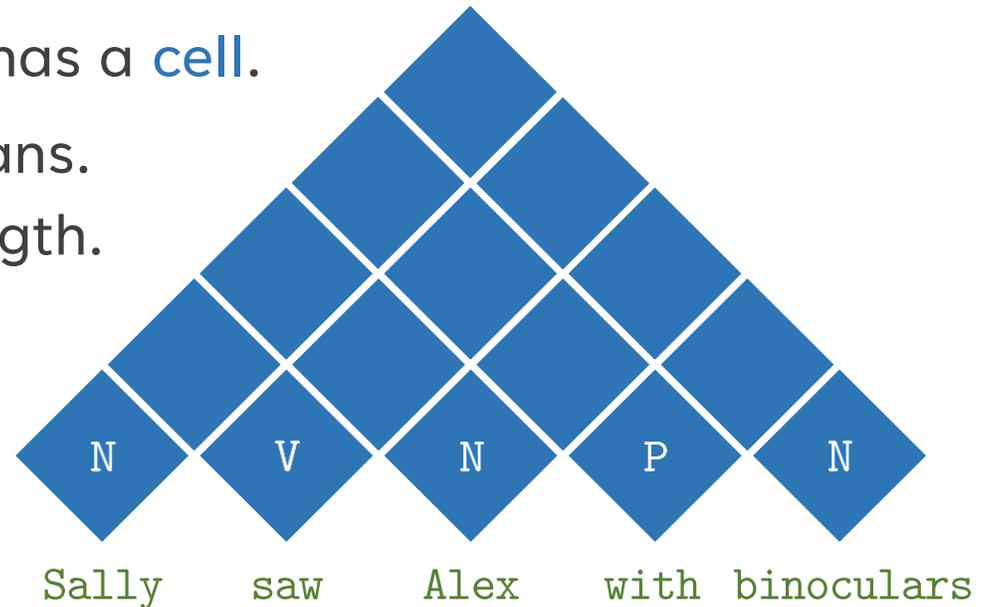
S	->	NP	VP	V	->	'saw'
VP	->	V	NP	P	->	'with'
VP	->	VP	PP	N	->	'binoculars'
PP	->	P	NP	N	->	'Sally'
NP	->	NP	PP	N	->	'Alex'
NP	->	N				



CKY PARSING

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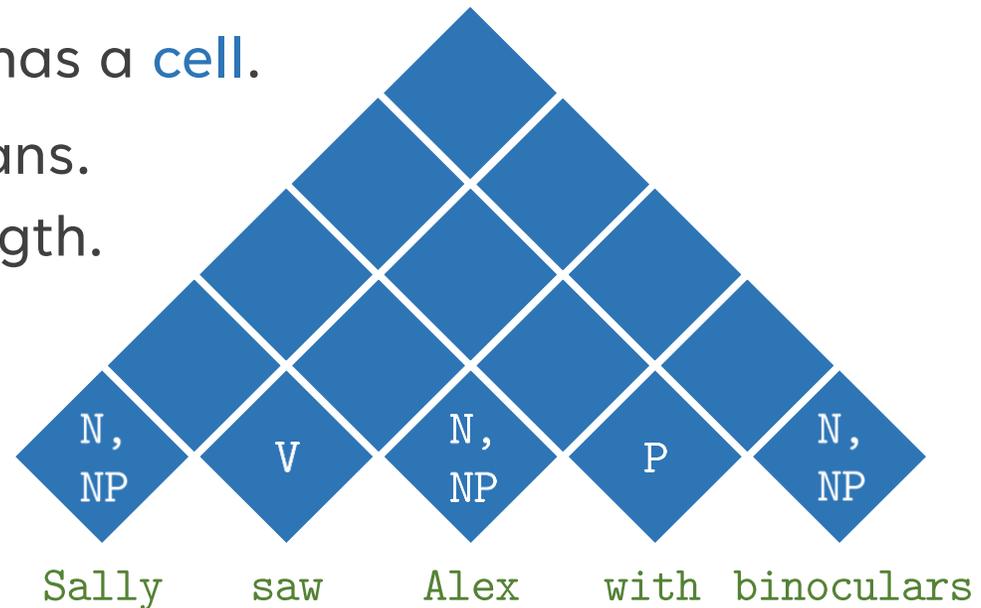
```
S  -> NP VP      V -> 'saw'
VP ->  V NP      P -> 'with'
VP ->  VP PP     N -> 'binoculars'
PP ->  P NP      N -> 'Sally'
NP ->  NP PP     N -> 'Alex'
NP ->  N
```



CKY PARSING

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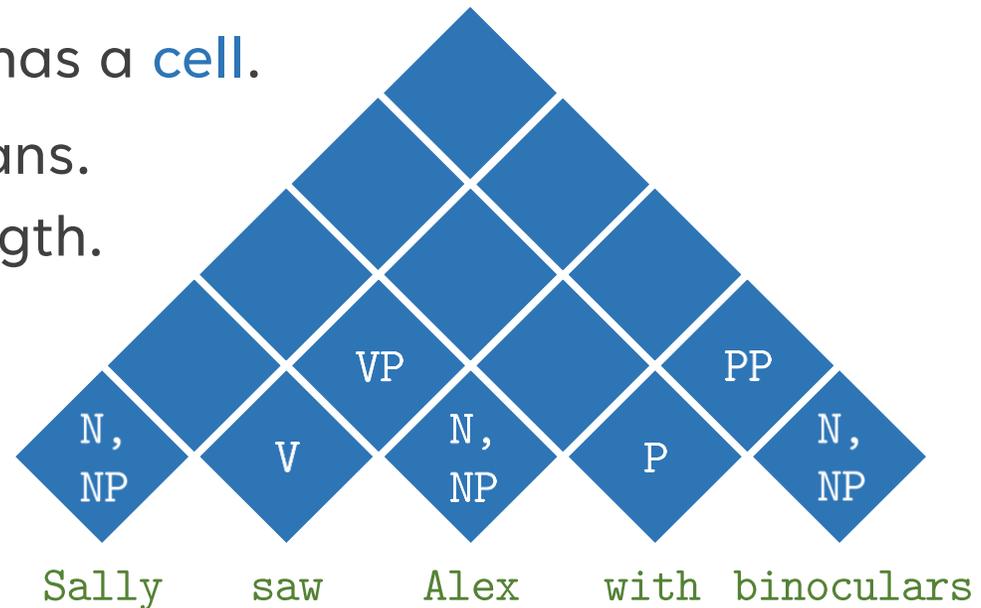
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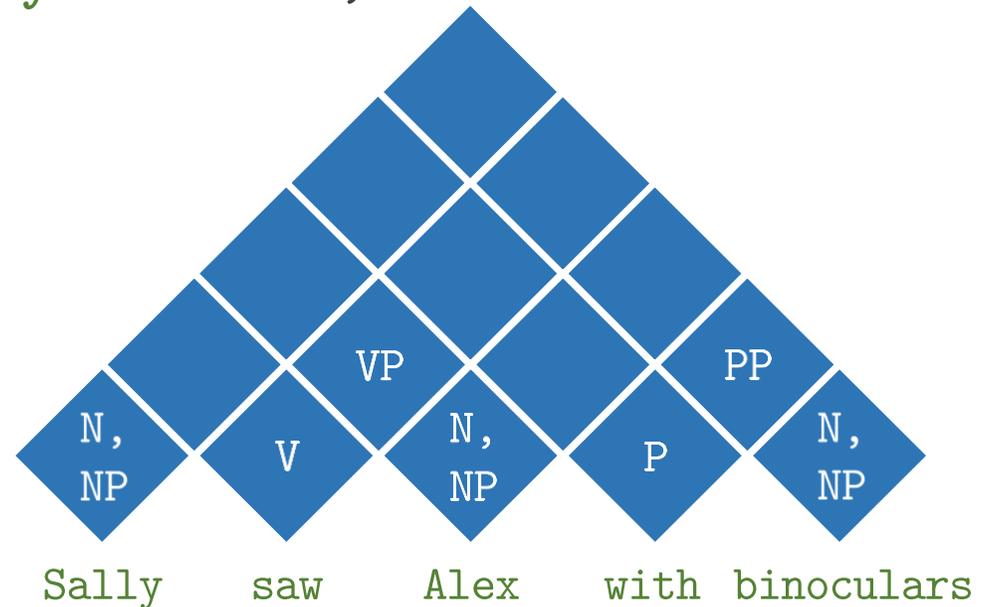
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VP	->	VP	PP	N	->	'binoculars'
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CKY PARSING

- For each cell (i, j) , we have to consider all pairs of cells (i, k) , (k, j) for all k such that $i \leq k \leq j$.
- For example, for the cell with 'Sally saw Alex,'

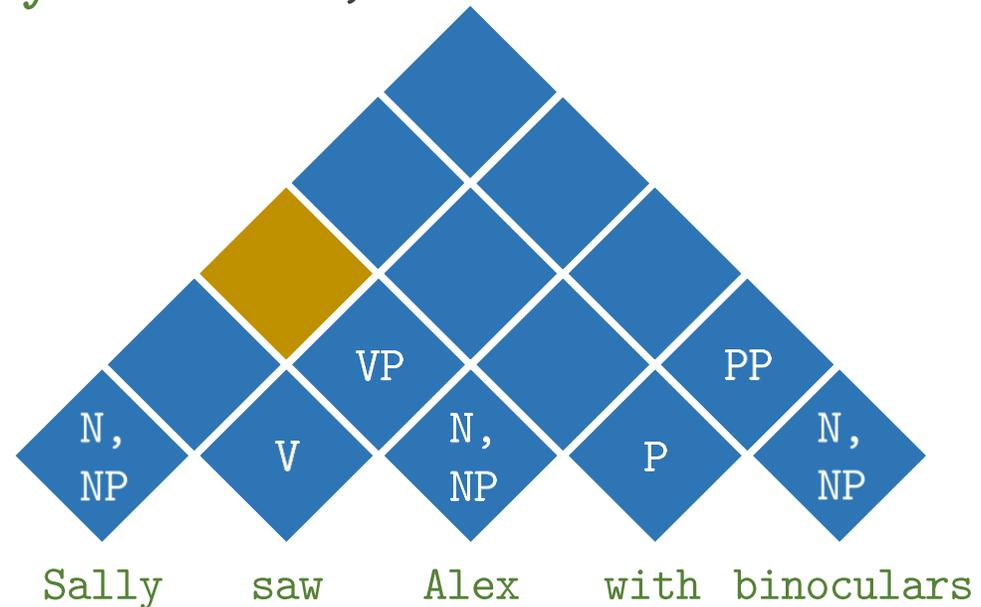
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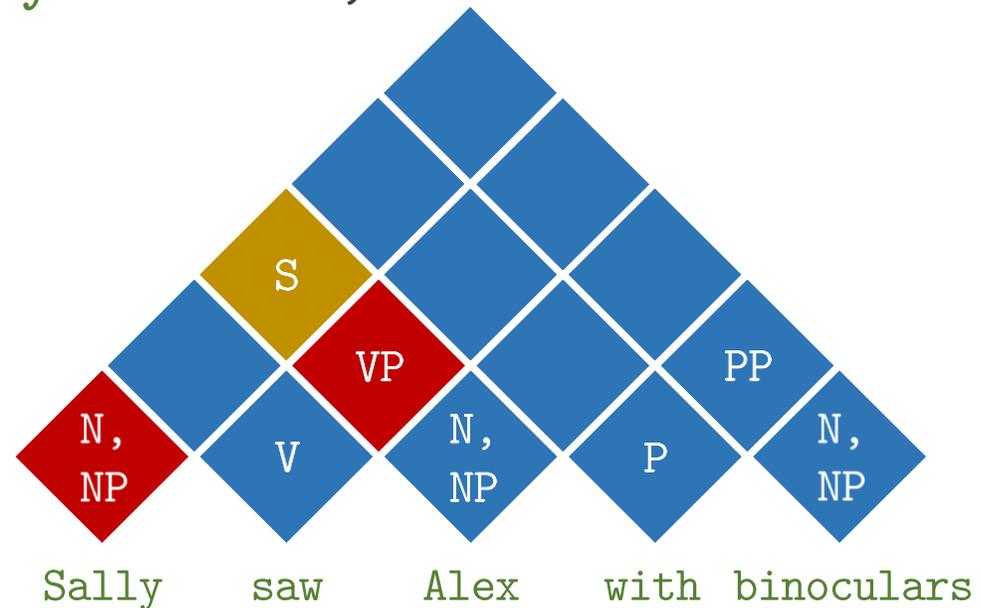
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NP	->	NP	PP	N	->	'Alex'
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CKY PARSING

- For each cell (i, j) , we have to consider all pairs of cells $(i, k), (k, j)$ for all k such that $i \leq k \leq j$.
- For example, for the cell with 'Sally saw Alex,'
- We must consider combining both:
 - 'Sally' and 'saw Alex'

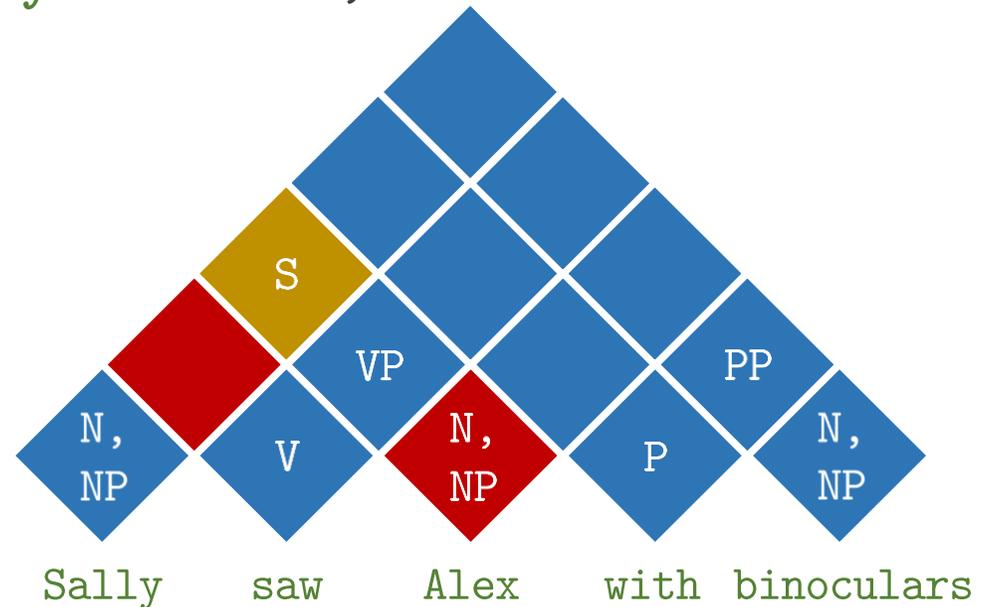
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VP	->	V	NP	P	->	'with'
VP	->	VP	PP	N	->	'binoculars'
PP	->	P	NP	N	->	'Sally'
NP	->	NP	PP	N	->	'Alex'
NP	->	N				



CKY PARSING

- For each cell (i, j) , we have to consider all pairs of cells $(i, k), (k, j)$ for all k such that $i \leq k \leq j$.
- For example, for the cell with 'Sally saw Alex,'
- We must consider combining both:
 - 'Sally' and 'saw Alex'
 - 'Sally saw' and 'Alex'

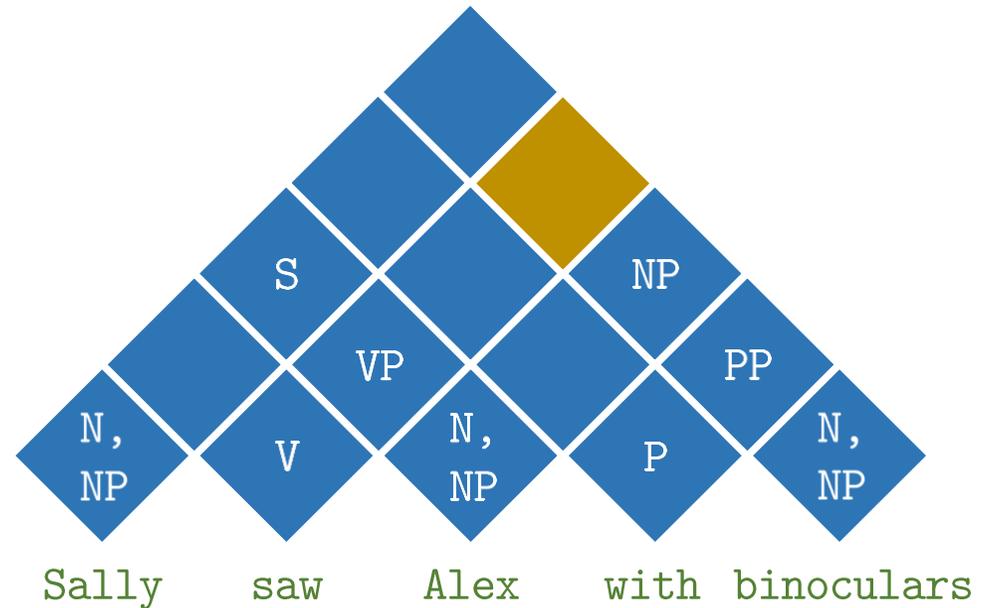
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CKY PARSING

- Similarly for the span ‘`saw Alex with binoculars`’ we must consider:

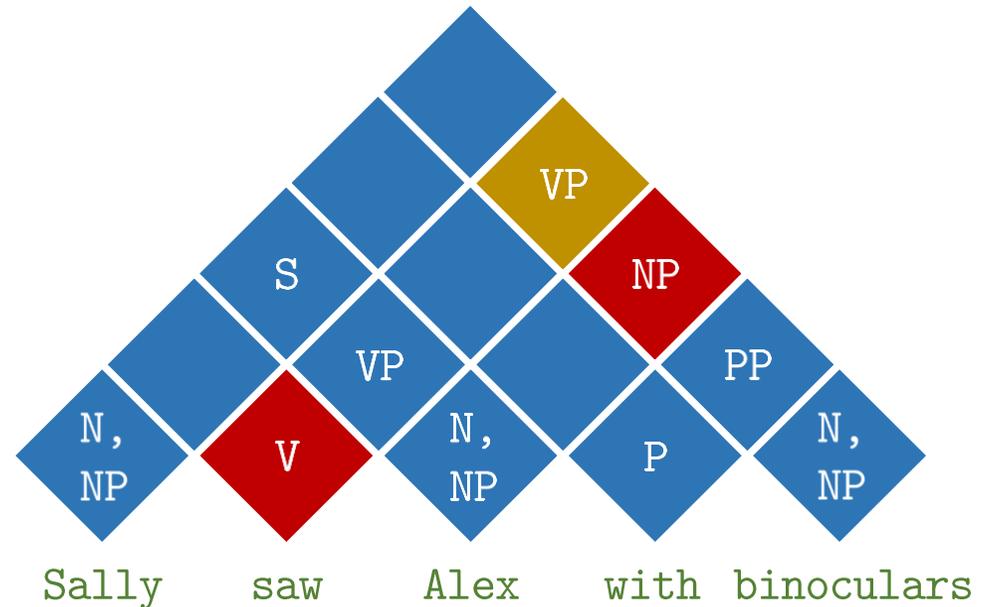
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VP	->	V	NP	P	->	‘with’
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NP	->	N				



CKY PARSING

- Similarly for the span ‘saw Alex with binoculars’ we must consider:
 - ‘saw’ and ‘Alex with binoculars’

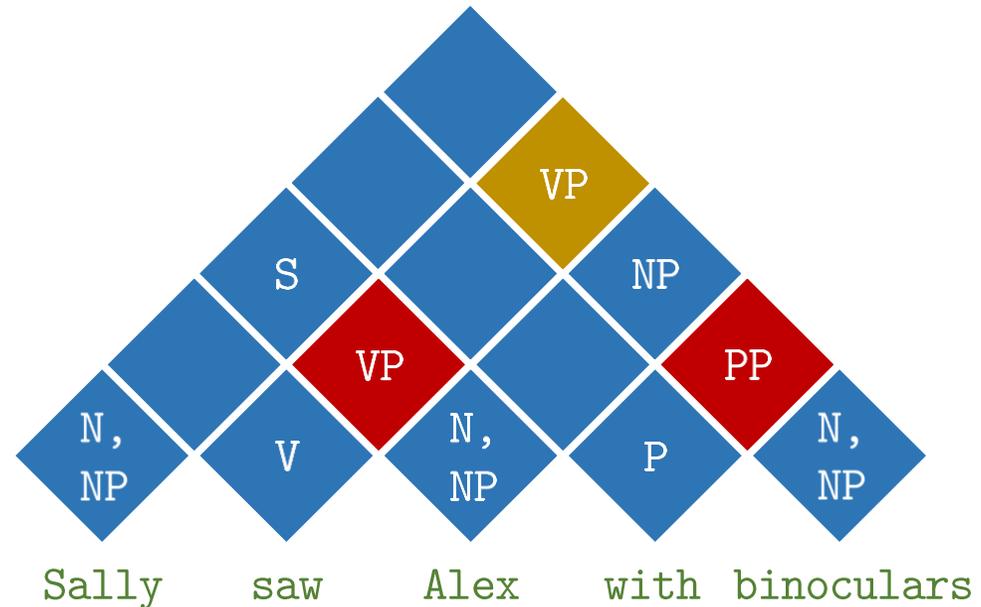
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CKY PARSING

- Similarly for the span ‘saw Alex with binoculars’ we must consider:
 - ‘saw’ and ‘Alex with binoculars’
 - ‘saw Alex’ and ‘with binoculars’

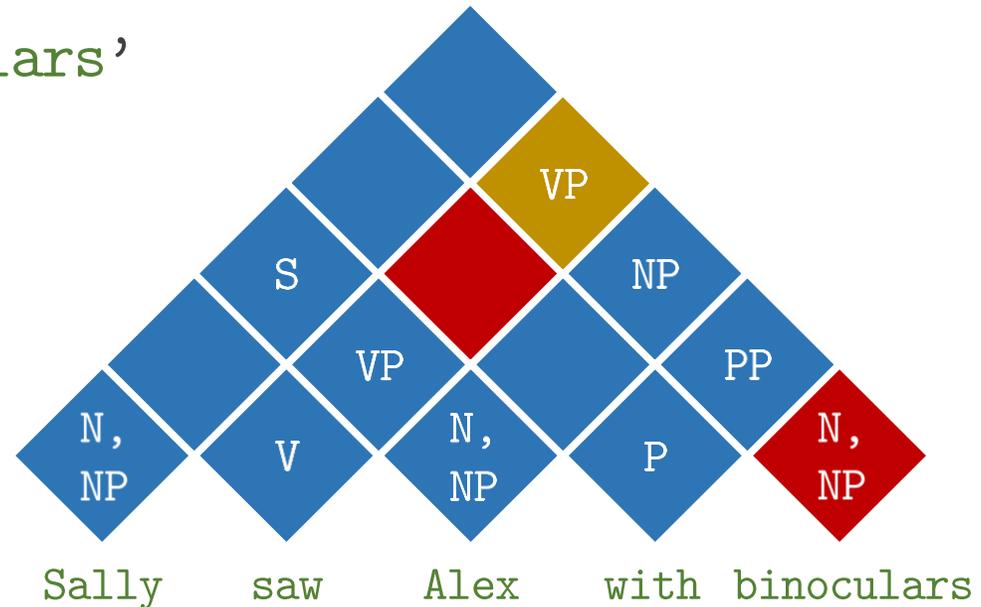
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VP	->	V	NP	P	->	‘with’
VP	->	VP	PP	N	->	‘binoculars’
PP	->	P	NP	N	->	‘Sally’
NP	->	NP	PP	N	->	‘Alex’
NP	->	N				



CKY PARSING

- Similarly for the span ‘saw Alex with binoculars’ we must consider:
 - ‘saw’ and ‘Alex with binoculars’
 - ‘saw Alex’ and ‘with binoculars’
 - ‘saw Alex with’ and ‘binoculars’

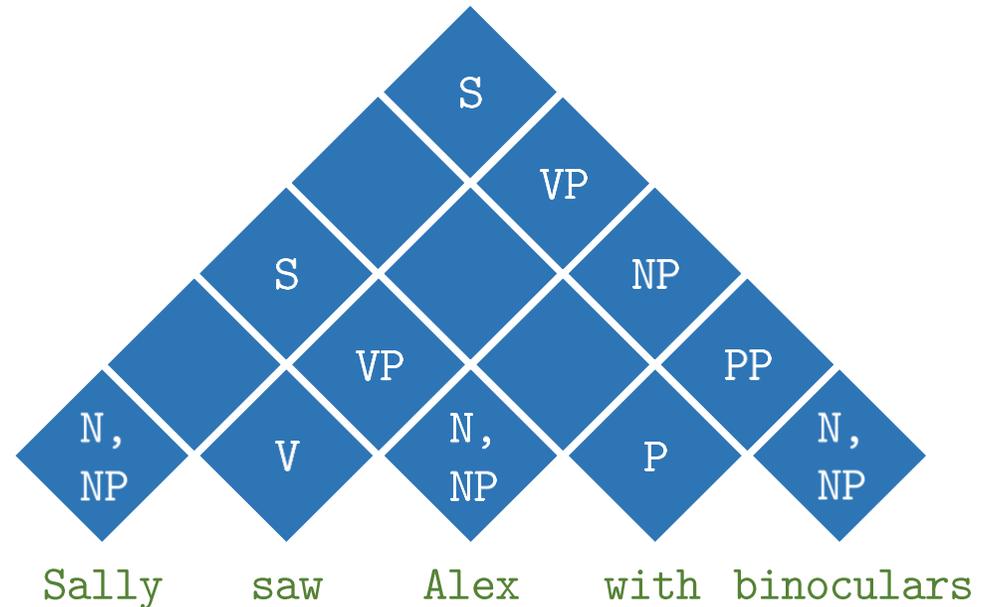
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VP	->	V	NP	P	->	‘with’
VP	->	VP	PP	N	->	‘binoculars’
PP	->	P	NP	N	->	‘Sally’
NP	->	NP	PP	N	->	‘Alex’
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CKY PARSING

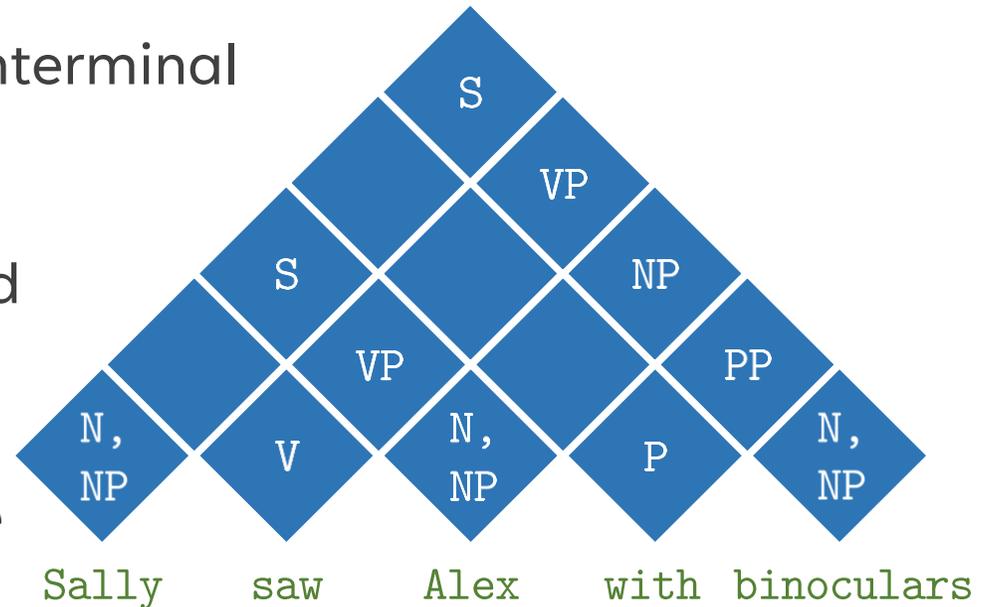
- We continue until we have processed all cells in the chart.
- In its simplest form, each cell simply records which nonterminals were parsed for that corresponding span.

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VP	->	V	NP	P	->	'with'
VP	->	VP	PP	N	->	'binoculars'
PP	->	P	NP	N	->	'Sally'
NP	->	NP	PP	N	->	'Alex'
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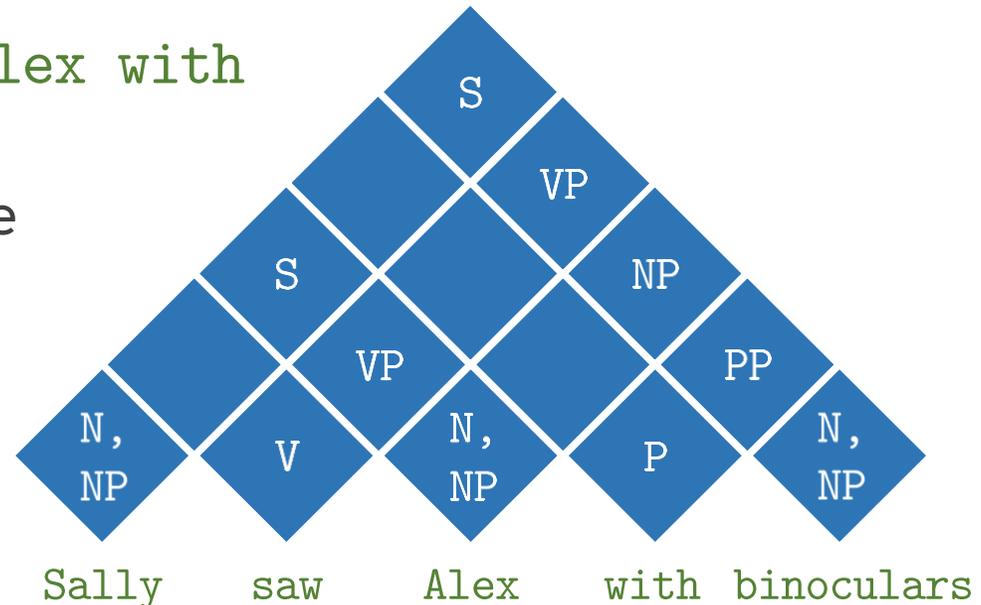
CKY PARSING

- We continue until we have processed all cells in the chart.
- In its simplest form, each cell simply records which nonterminals were parsed for that corresponding span.
- If instead, we record *how* each nonterminal was constructed in each cell,
- E.g., the VP for the span 'saw Alex with binoculars' was constructed from the V 'saw' and the NP 'Alex with binoculars,'
- We can reconstruct the syntax tree by following the pointers from the root of the chart.



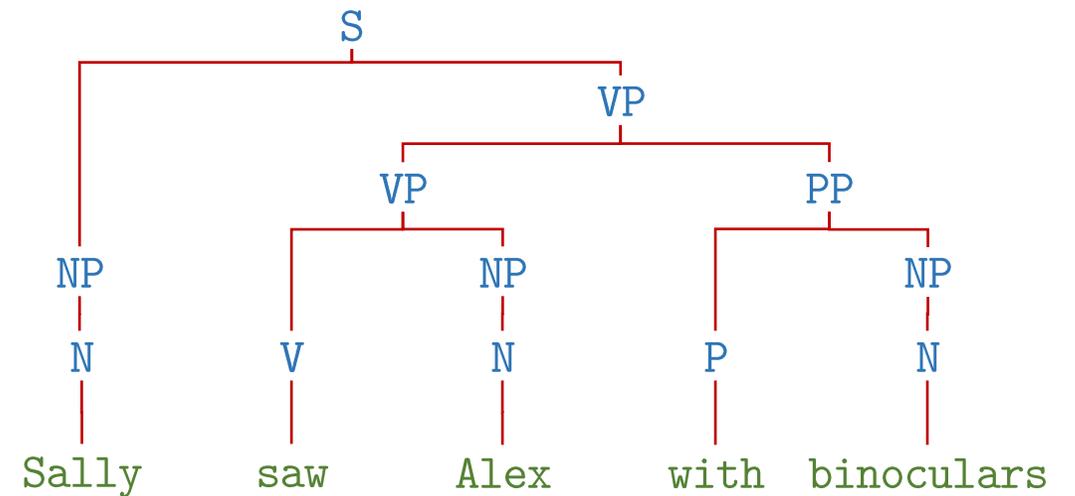
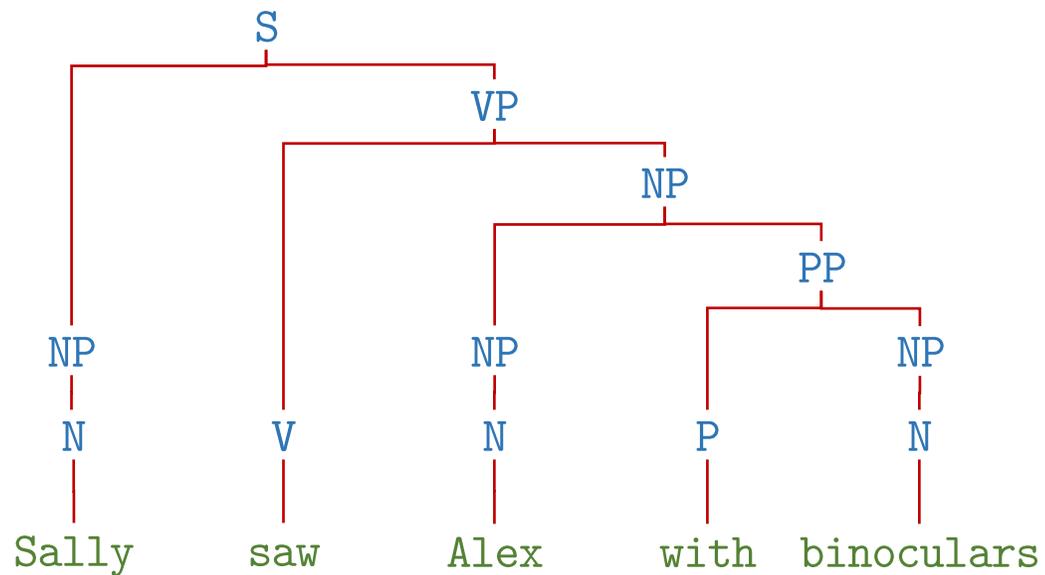
CKY PARSING

- CKY can also parse **ambiguous sentences**.
- Here, the **VP** for the span 'saw Alex with binoculars' can be constructed in two ways:
 1. from the **V** 'saw' and the **NP** 'Alex with binoculars,'
 2. from the **VP** 'saw Alex' and the **PP** 'with binoculars.'
- Both these constructions must be stored in the cell for 'saw Alex with binoculars.'



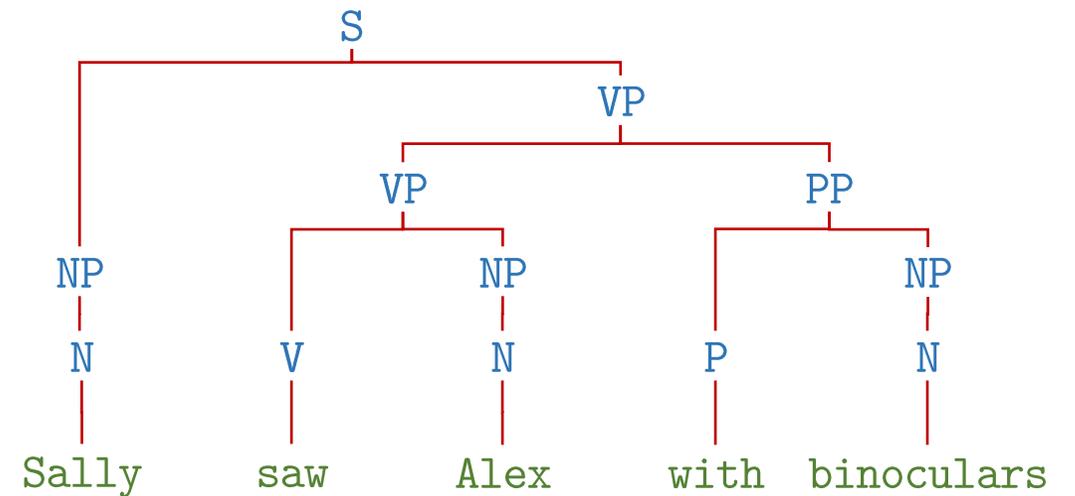
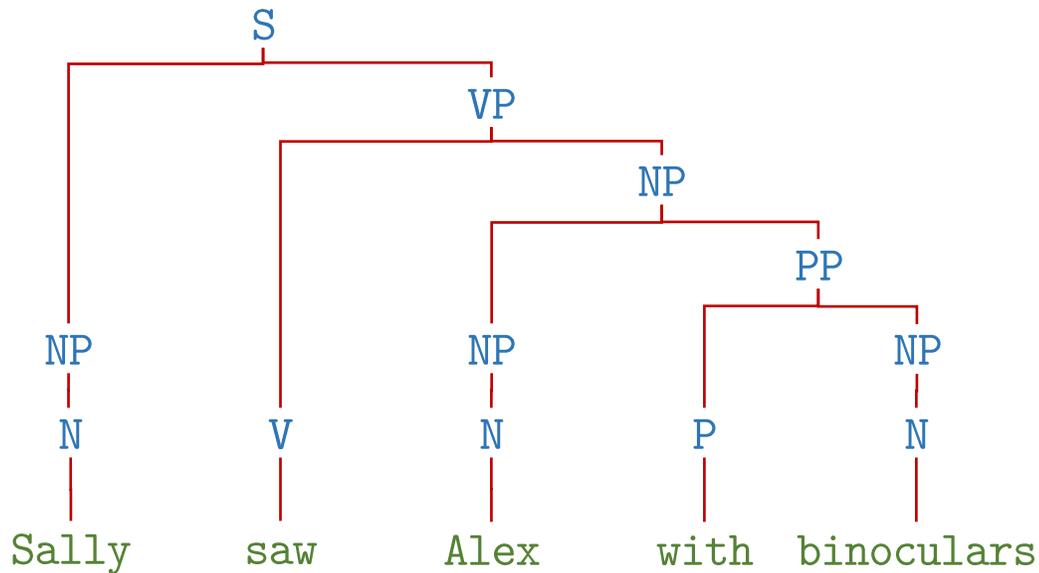
SYNTACTIC AMBIGUITY

- Depending on which of the two ambiguous constructions we choose when reconstructing the syntax tree at that cell,
- We can obtain two different syntax trees:



SYNTACTIC AMBIGUITY

- These ambiguous syntax trees correspond to two different interpretations:
 1. Alex has the binoculars, and Sally sees Alex.
 2. Sally uses binoculars to see Alex.



CKY RUNNING TIME

- What is the running time of CKY parsing?
- For each cell (i, j) , we had to consider all pairs of cells (i, k) , (k, j) for all k such that $i \leq k \leq j$.
 - For each i, j, k , we iterate over each rule in the grammar to check for a match.
- We have (roughly half of) n^2 cells, where n is the length of the sentence.
- In the worst case, there are n possible values of k .
- Thus the running time is $O(|G|n^3)$,
 - Where $|G|$ is the number of production rules in the grammar G .

PARSING CFGS

- Another dynamic programming approach to CFG parsing was developed by Earley (1968, 1970).
 - Called **Earley parsing**.
- Unlike CKY which is a bottom-up parsing approach, Earley is top-down.
 - I.e., we start from the root of the syntax tree and work our way down to the leaves/terminals.

EARLEY PARSING

- Consider the example: ‘Sally saw Alex with binoculars’
- We start with the root nonterminal **S**.
 - Create an initial state for any rule of the form **S** → ...:
- The dot ‘.’ indicates the current position of the parser.
 - In this example state, we haven’t parsed anything yet.
- Push this state onto a queue.

S → NP VP	V → ‘saw’
VP → V NP	P → ‘with’
VP → VP PP	N → ‘binoculars’
PP → P NP	N → ‘Sally’
NP → NP PP	N → ‘Alex’
NP → N	

queue

S → . NP VP
i=0, k=0

EARLEY PARSING

- Consider the example: ‘Sally saw Alex with binoculars’
- Repeat the following:
 - Pop a state from the queue.
 - If the symbol following the ‘.’ is a nonterminal, do a **prediction** step.
 - Create a new state for any production rule of the form NP → ...
 - The old state is added to a list of “waiting” states.

S → NP VP	V → ‘saw’
VP → V NP	P → ‘with’
VP → VP PP	N → ‘binoculars’
PP → P NP	N → ‘Sally’
NP → NP PP	N → ‘Alex’
NP → N	

S → . NP VP
i=0, k=0

queue

NP → . N
i=0, k=0

NP → . NP PP
i=0, k=0

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat the following:
 - Pop a state from the queue.
 - We avoid repeating prediction steps for the same nonterminal at the same position (i.e., NP at position 0).
 - Otherwise we would have an infinite loop.

```
S -> NP VP      V -> 'saw'
VP -> V NP       P -> 'with'
VP -> VP PP      N -> 'binoculars'
PP -> P NP       N -> 'Sally'
NP -> NP PP      N -> 'Alex'
NP -> N
```

```
NP -> . NP PP
i=0, k=0
```

```
NP -> . N
i=0, k=0
```

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat the following:
 - Pop a state from the queue.
 - If the symbol following the '.' is a nonterminal, do a **prediction** step.
 - Create a new state for any production rule of the form $N \rightarrow \dots$

S	->	NP	VP	V	->	'saw'
VP	->	V	NP	P	->	'with'
VP	->	VP	PP	N	->	'binoculars'
PP	->	P	NP	N	->	'Sally'
NP	->	NP	PP	N	->	'Alex'
NP	->	N				

NP -> . N
i=0, k=0

queue

N -> . 'Sally'
i=0, k=0

N -> . 'Alex'
i=0, k=0

N -> . 'binoculars'
i=0, k=0

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat the following:
 - Pop a state from the queue.
 - If the symbol following '.' is a terminal, do a **scanning** step.
 - If the token at position **k** in the sentence matches the terminal, push a new state where the '.' moves forward.

S	->	NP	VP	V	->	'saw'
VP	->	V	NP	P	->	'with'
VP	->	VP	PP	N	->	'binoculars'
PP	->	P	NP	N	->	'Sally'
NP	->	NP	PP	N	->	'Alex'
NP	->	N				

```
N -> . 'binoculars'  
i=0, k=0
```

queue

```
N -> . 'Sally'  
i=0, k=0
```

```
N -> . 'Alex'  
i=0, k=0
```

EARLEY PARSING

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VP	->	VP	PP	N	->	'binoculars'
PP	->	P	NP	N	->	'Sally'
NP	->	NP	PP	N	->	'Alex'
NP	->	N				

N -> . 'Alex'
i=0, k=0

N -> . 'Sally'
i=0, k=0

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat the following:
 - Pop a state from the queue.
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S → NP VP V → 'saw'
VP → V NP P → 'with'
VP → VP PP N → 'binoculars'
PP → P NP N → 'Sally'
NP → NP PP N → 'Alex'
NP → N

N → . 'Sally'
i=0, k=0

N → 'Sally' .
i=0, k=1

queue

EARLEY PARSING

- Consider the example: ‘Sally saw Alex with binoculars’
- Repeat the following:
 - Pop a state from the queue.
 - If the ‘.’ is at the end of the rule, do a **completion** step.
 - For all “waiting” states where an **N** follows the ‘.’, push a new state where the ‘.’ moves forward.
- Here, we have successfully parsed **N** at span (0,1).

S	->	NP VP	V	->	‘saw’
VP	->	V NP	P	->	‘with’
VP	->	VP PP	N	->	‘binoculars’
PP	->	P NP	N	->	‘Sally’
NP	->	NP PP	N	->	‘Alex’
NP	->	N			

N -> ‘Sally’ .
i=0, k=1

NP -> N .
i=0, k=1

queue

EARLEY PARSING

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- Repeat the following:
 - Pop a state from the queue.
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 - For all “waiting” states where an **N** follows the ‘.’, push a new state where the ‘.’ moves forward.
- Here, we have successfully parsed **NP** at span (0,1).

S → NP VP	V → ‘saw’
VP → V NP	P → ‘with’
VP → VP PP	N → ‘binoculars’
PP → P NP	N → ‘Sally’
NP → NP PP	N → ‘Alex’
NP → N	

NP → N .
i=0, k=1

S → NP . VP
i=0, k=1

queue

EARLEY PARSING

- Consider the example: ‘Sally saw Alex with binoculars’
- Repeat the following:
 - Pop a state from the queue.
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PP → P NP	N → ‘Sally’
NP → NP PP	N → ‘Alex’
NP → N	

S → NP . VP
i=0, k=1

queue

VP → . VP PP
i=1, k=1

VP → . V NP
i=1, k=1

EARLEY PARSING

- Consider the example: ‘Sally saw Alex with binoculars’
- Repeat the following:
 - Pop a state from the queue.
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VP	->	V	NP	P	->	‘with’
VP	->	VP	PP	N	->	‘binoculars’
PP	->	P	NP	N	->	‘Sally’
NP	->	NP	PP	N	->	‘Alex’
NP	->	N				

VP -> . V NP
i=1, k=1

queue

V -> . ‘saw’
i=1, k=1

VP -> . VP PP
i=1, k=1

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$VP \rightarrow \cdot VP PP$
 $i=1, k=1$

$V \rightarrow \cdot \text{'saw'}$
 $i=1, k=1$

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S \rightarrow NP VP
VP \rightarrow V NP
VP \rightarrow VP PP
PP \rightarrow P NP
NP \rightarrow NP PP
NP \rightarrow N

V \rightarrow 'saw'
P \rightarrow 'with'
N \rightarrow 'binoculars'
N \rightarrow 'Sally'
N \rightarrow 'Alex'

V \rightarrow . 'saw'
i=1, k=1

V \rightarrow 'saw' .
i=1, k=2

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S	\rightarrow	NP	VP	V	\rightarrow	'saw'
VP	\rightarrow	V	NP	P	\rightarrow	'with'
VP	\rightarrow	VP	PP	N	\rightarrow	'binoculars'
PP	\rightarrow	P	NP	N	\rightarrow	'Sally'
NP	\rightarrow	NP	PP	N	\rightarrow	'Alex'
NP	\rightarrow	N				

V \rightarrow 'saw' .
i=1, k=2

VP \rightarrow V . NP
i=1, k=2

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
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$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
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$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$VP \rightarrow V . NP$
 $i=1, k=2$

queue

$NP \rightarrow . N$
 $i=2, k=2$

$NP \rightarrow . NP PP$
 $i=2, k=2$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$NP \rightarrow \cdot NP PP$
 $i=2, k=2$

$NP \rightarrow \cdot N$
 $i=2, k=2$

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$NP \rightarrow . N$
 $i=2, k=2$

queue

$N \rightarrow . \text{'Sally'}$
 $i=2, k=2$

$N \rightarrow . \text{'Alex'}$
 $i=2, k=2$

$N \rightarrow . \text{'binoculars'}$
 $i=2, k=2$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$N \rightarrow . \text{'binoculars'}$
 $i=2, k=2$

queue

$N \rightarrow . \text{'Sally'}$
 $i=2, k=2$

$N \rightarrow . \text{'Alex'}$
 $i=2, k=2$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$N \rightarrow \cdot \text{'Alex'}$
 $i=2, k=2$

$N \rightarrow \text{'Alex'} \cdot$
 $i=2, k=3$

$N \rightarrow \cdot \text{'Sally'}$
 $i=2, k=2$

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S \rightarrow NP VP
VP \rightarrow V NP
VP \rightarrow VP PP
PP \rightarrow P NP
NP \rightarrow NP PP
NP \rightarrow N

V \rightarrow 'saw'
P \rightarrow 'with'
N \rightarrow 'binoculars'
N \rightarrow 'Sally'
N \rightarrow 'Alex'

N \rightarrow . 'Sally'
i=2, k=2

N \rightarrow 'Alex' .
i=2, k=3

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S	\rightarrow	NP	VP	V	\rightarrow	'saw'
VP	\rightarrow	V	NP	P	\rightarrow	'with'
VP	\rightarrow	VP	PP	N	\rightarrow	'binoculars'
PP	\rightarrow	P	NP	N	\rightarrow	'Sally'
NP	\rightarrow	NP	PP	N	\rightarrow	'Alex'
NP	\rightarrow	N				

N \rightarrow 'Alex' .
i=2, k=3

NP \rightarrow N .
i=2, k=3

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$NP \rightarrow N \cdot$
 $i=2, k=3$

$NP \rightarrow NP \cdot PP$
 $i=2, k=3$

$VP \rightarrow V NP \cdot$
 $i=1, k=3$

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$VP \rightarrow V NP .$
 $i=1, k=3$

queue

$VP \rightarrow VP . PP$
 $i=1, k=3$

$S \rightarrow NP VP .$
 $i=0, k=3$

$NP \rightarrow NP . PP$
 $i=2, k=3$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$NP \rightarrow NP . PP$
 $i=2, k=3$

queue

$PP \rightarrow . P NP$
 $i=3, k=3$

$VP \rightarrow VP . PP$
 $i=1, k=3$

$S \rightarrow NP VP .$
 $i=0, k=3$

EARLEY PARSING

- Consider the example: ‘Sally saw Alex with binoculars’
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.
- Here we successfully parsed S ,
 - But we didn’t reach the end of the sentence,
 - So we continue.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$S \rightarrow NP VP .$
 $i=0, k=3$

queue

$PP \rightarrow . P NP$
 $i=3, k=3$

$VP \rightarrow VP . PP$
 $i=1, k=3$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$VP \rightarrow VP . PP$
 $i=1, k=3$

$PP \rightarrow . P NP$
 $i=3, k=3$

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$PP \rightarrow . P NP$
 $i=3, k=3$

$P \rightarrow . \text{'with'}$
 $i=3, k=3$

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$P \rightarrow \cdot \text{'with'}$
 $i=3, k=3$

$P \rightarrow \text{'with'} \cdot$
 $i=3, k=4$

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S \rightarrow NP VP
VP \rightarrow V NP
VP \rightarrow VP PP
PP \rightarrow P NP
NP \rightarrow NP PP
NP \rightarrow N

V \rightarrow 'saw'
P \rightarrow 'with'
N \rightarrow 'binoculars'
N \rightarrow 'Sally'
N \rightarrow 'Alex'

P \rightarrow 'with' .
i=3, k=4

PP \rightarrow P . NP
i=3, k=4

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$PP \rightarrow P . NP$
 $i=3, k=4$

queue

$NP \rightarrow . N$
 $i=4, k=4$

$NP \rightarrow . NP PP$
 $i=4, k=4$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S	\rightarrow	NP	VP	V	\rightarrow	'saw'
VP	\rightarrow	V	NP	P	\rightarrow	'with'
VP	\rightarrow	VP	PP	N	\rightarrow	'binoculars'
PP	\rightarrow	P	NP	N	\rightarrow	'Sally'
NP	\rightarrow	NP	PP	N	\rightarrow	'Alex'
NP	\rightarrow	N				

NP \rightarrow . NP PP
i=4, k=4

NP \rightarrow . N
i=4, k=4

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$NP \rightarrow . N$
 $i=4, k=4$

queue

$N \rightarrow . \text{'Sally'}$
 $i=4, k=4$

$N \rightarrow . \text{'Alex'}$
 $i=4, k=4$

$N \rightarrow . \text{'binoculars'}$
 $i=4, k=4$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$N \rightarrow . \text{'binoculars'}$
 $i=4, k=4$

queue

$N \rightarrow \text{'binoculars'}$.
 $i=4, k=5$

$N \rightarrow . \text{'Sally'}$
 $i=4, k=4$

$N \rightarrow . \text{'Alex'}$
 $i=4, k=4$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$N \rightarrow \cdot \text{'Alex'}$
 $i=4, k=4$

queue

$N \rightarrow \text{'binoculars'} \cdot$
 $i=4, k=5$

$N \rightarrow \cdot \text{'Sally'}$
 $i=4, k=4$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S	\rightarrow	NP	VP	V	\rightarrow	'saw'
VP	\rightarrow	V	NP	P	\rightarrow	'with'
VP	\rightarrow	VP	PP	N	\rightarrow	'binoculars'
PP	\rightarrow	P	NP	N	\rightarrow	'Sally'
NP	\rightarrow	NP	PP	N	\rightarrow	'Alex'
NP	\rightarrow	N				

N \rightarrow . 'Sally'
i=4, k=4

N \rightarrow 'binoculars' .
i=4, k=5

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S \rightarrow NP VP
VP \rightarrow V NP
VP \rightarrow VP PP
PP \rightarrow P NP
NP \rightarrow NP PP
NP \rightarrow N

V \rightarrow 'saw'
P \rightarrow 'with'
N \rightarrow 'binoculars'
N \rightarrow 'Sally'
N \rightarrow 'Alex'

N \rightarrow 'binoculars' .
i=4, k=5

NP \rightarrow N .
i=4, k=5

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

S \rightarrow NP VP
VP \rightarrow V NP
VP \rightarrow VP PP
PP \rightarrow P NP
NP \rightarrow NP PP
NP \rightarrow N

V \rightarrow 'saw'
P \rightarrow 'with'
N \rightarrow 'binoculars'
N \rightarrow 'Sally'
N \rightarrow 'Alex'

NP \rightarrow N .
i=4, k=5

PP \rightarrow P NP .
i=3, k=5

queue

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$PP \rightarrow P NP \cdot$
 $i=3, k=5$

queue

$VP \rightarrow VP PP \cdot$
 $i=1, k=5$

$NP \rightarrow NP PP \cdot$
 $i=2, k=5$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$NP \rightarrow NP PP .$
 $i=2, k=5$

queue

$VP \rightarrow V NP .$
 $i=1, k=5$

$VP \rightarrow VP PP .$
 $i=1, k=5$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$VP \rightarrow VP PP .$
 $i=1, k=5$

queue

$S \rightarrow NP VP .$
 $i=0, k=5$

$VP \rightarrow V NP .$
 $i=1, k=5$

EARLEY PARSING

- Consider the example: 'Sally saw Alex with binoculars'
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$VP \rightarrow V NP \cdot$
 $i=1, k=5$

queue

$S \rightarrow NP VP \cdot$
 $i=0, k=5$

$S \rightarrow NP VP \cdot$
 $i=0, k=5$

EARLEY PARSING

- Consider the example: ‘Sally saw Alex with binoculars’
- Repeat until we complete a rule of the form $S \rightarrow \dots$
 - Depending on the popped state, we perform either prediction, scanning, or completion steps.
- We have found a valid parse of S for the full sentence.
- If instead, the queue became empty, then the input sentence is not part of the language.

$S \rightarrow NP VP$	$V \rightarrow \text{'saw'}$
$VP \rightarrow V NP$	$P \rightarrow \text{'with'}$
$VP \rightarrow VP PP$	$N \rightarrow \text{'binoculars'}$
$PP \rightarrow P NP$	$N \rightarrow \text{'Sally'}$
$NP \rightarrow NP PP$	$N \rightarrow \text{'Alex'}$
$NP \rightarrow N$	

$S \rightarrow NP VP .$
 $i=0, k=5$

$S \rightarrow NP VP .$
 $i=0, k=5$

queue

EARLEY PARSING

- If, for each state, we keep track of substates that completed it,
 - E.g., in the state $S \rightarrow NP VP \cdot$, we keep a pointer to the completed state for NP , and another for VP ,
 - We can reconstruct the syntax tree by following the backpointers from the completed state for S .
 - Similar to CKY.
- Unlike CKY, Earley can be applied to **any CFG**,
 - Including those not in Chomsky normal form.

EARLEY VS CKY

- Since Earley is a top-down parser, it can avoid producing **phantom parses**.
 - These are parses that are useless in the final parse.
 - E.g., ‘It traveled 90% of the speed of light.’
 - ‘speed’ and ‘light’ are nouns here,
 - But ‘speed’ can be a verb (e.g., ‘Don’t speed on the highway.’).
 - And ‘light’ can be an adjective (e.g., ‘the light jacket’).
 - CKY would produce all valid parses of these phrases (noun, verb, and adjective phrases).
 - But Earley would only produce the phrases that are valid in the context of the whole sentence.
 - In this example, ‘speed’ and ‘light’ would only be parsed as nouns.

EARLEY RUNNING TIME

- Is Earley faster than CKY?
- In the worst case, Earley would also require $O(|G|n^3)$ time.
- However, for unambiguous grammars, Earley runs in $O(|G|n^2)$.
- There is a smaller class of simpler CFGs called deterministic CFGs on which Earley can run in $O(|G|n)$ time.

GENERALIZING CKY AND EARLEY

- CKY and Earley parsers are both dynamic programming solutions to the problem of parsing CFGs.
- But is there a way to view these algorithms as instances of the same framework?
- Consider the main loops of each algorithm:
 - In CKY, we loop over all spans (i, j) in order of increasing $j - i$.
 - So each state is a span (i, j) ,
 - We consider all subspans (i, k) , (k, j) such that k is between i and j .
 - And check whether we can construct a parse tree from the subtrees in (i, k) and (k, j) .

GENERALIZING CKY AND EARLEY

- CKY and Earley parsers are both dynamic programming solutions to the problem of parsing CFGs.
- But is there a way to view these algorithms as instances of the same framework?
- Consider the main loops of each algorithm:
 - In CKY, we loop over all spans (i, j, A) in order of increasing $j - i$.
 - So each state contains a span (i, j) and nonterminal A ,
 - We consider all subspans $(i, k), (k, j)$ such that k is between i and j .
 - And check whether we can construct a parse tree rooted at A from the subtrees in (i, k) and (k, j) .

GENERALIZING CKY AND EARLEY

- CKY and Earley parsers are both dynamic programming solutions to the problem of parsing CFGs.
- But is there a way to view these algorithms as instances of the same framework?
- Consider the main loops of each algorithm:
 - In Earley, we loop over states of the form $A \rightarrow B_1 \dots B_m \cdot B_{m+1} \dots B_n$ with start position i and current position k .
 - The next step depends on the symbol following the ‘.’:
 - If B_{m+1} is a nonterminal, we do a **prediction** step.
 - I.e., create a new state for all rules of the form $B_{m+1} \rightarrow \dots$

GENERALIZING CKY AND EARLEY

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- Consider the main loops of each algorithm:
 - In Earley, we loop over states of the form $A \rightarrow B_1 \dots B_m \cdot B_{m+1} \dots B_n$ with start position i and current position k .
 - The next step depends on the symbol following the ‘.’:
 - If B_{m+1} is a terminal, we do a **scanning** step.
 - I.e., check if the input sentence matches the terminal B_{m+1} at position k .
 - If so, create a new state $A \rightarrow B_1 \dots B_{m+1} \cdot \dots B_n$ with incremented k .

GENERALIZING CKY AND EARLEY

- CKY and Earley parsers are both dynamic programming solutions to the problem of parsing CFGs.
- But is there a way to view these algorithms as instances of the same framework?
- Consider the main loops of each algorithm:
 - In Earley, we loop over states of the form $A \rightarrow B_1 \dots B_m \cdot$ with start position i and current position k .
 - The next step depends on the symbol following the ‘.’:
 - If there is no symbol after ‘.’, do a **completion** step.
 - For every state “waiting” for A at position i , create a new state where the dot moves forward and k is updated appropriately.

GENERALIZING CKY AND EARLEY

- CKY and Earley parsers are both dynamic programming solutions to the problem of parsing CFGs.
- But is there a way to view these algorithms as instances of the same framework?
- Consider the main loops of each algorithm.
 - In both CKY and Earley, we can imagine the states being added to a **priority queue**.
 - At each iteration, we pop one state from the queue and process it.
- The priority of a state is computed differently in CKY and Earley.
 - In CKY, states with **shorter spans** are prioritized.
 - How are states prioritized in Earley?

ORDER OF STATES IN EARLEY PARSING

- In Earley parsing, states are removed from the queue in the same order they were added.
 - We can replicate this behavior in a priority queue by setting the priority of the state to be the **iteration number**.
- But it's not necessary to follow this prioritization.
 - We can process states in a different order and still have a correct parsing algorithm.

ORDER OF STATES IN EARLEY PARSING

- But there is a minor caveat:
 - We assumed that whenever we have a **completion** step $A \rightarrow B_1 \dots B_m \cdot$ with start position i ,
 - All prediction steps of the form $C \rightarrow \dots \cdot A \dots$ have already been processed earlier in a **prediction** step.
- If we change the order of visited states, this may no longer be true.
 - But we can resolve this issue by adding an extra step during **prediction**.
 - Whenever we have a prediction step $C \rightarrow \dots \cdot A \dots$, we check if there are any completed parses for A at the same position.
 - If there are, then create a new state $C \rightarrow \dots A \cdot \dots$ with k updated accordingly.

ORDER OF STATES IN EARLEY PARSING

- There is also an **optimization** available here:
 - Whenever we do a prediction step, $C \rightarrow \dots . A \dots$ at position k , we create a new state for each rule of the form $A \rightarrow \dots$ with start position k .
 - But we don't need to do this more than once.
 - So we can avoid doing this multiple times by keeping track of whether we have “expanded” A at position k in the past.
 - If we have, then avoid “expanding” A at k again.

CKY VS EARLEY INITIALIZATION

- There is one more major difference between CKY and Earley parsing:
- What are the initial states in the priority queue before starting the main loop?
 - In CKY, we add an initial state for *every span* containing 1 token.
 - In Earley, we add an initial state for *every rule* of the form $S \rightarrow \dots$ where S is the root nonterminal.
- Is related to the bottom-up vs top-down approach of CKY and Earley parsing.

BRANCH AND BOUND

- So it seems like CKY and Earley parsing share a lot of structure.
 - Is there a unifying description?
- **Branch-and-bound** is a general class of algorithms for **discrete optimization/search**.
 - Say we want to find a target object x in a large set of objects S that maximizes some priority function $f(x)$.
 - For search, this can be a simple indicator function.
 - $f(x) = 1$ if and only if x is the object we're searching for.
 - First, we partition (i.e., “**branch**”) the set S into subsets:
 S_1, \dots, S_n
such that their union covers the full set: $S_1 \cup \dots \cup S_n = S$

BRANCH AND BOUND

- Next, for each subset S_i , create a new state and add it to the priority queue.
 - What should we set its priority to?
 - Ideally, it should be $\max_{x \in S_i} f(x)$.
 - But this quantity can be intractable to compute,
 - Especially if S_i is very large.
 - Instead, we can use an easy-to-compute an upper bound on this quantity:
$$g(S_i) \geq \max_{x \in S_i} f(x)$$
 - Just set the priority of the new state to $g(S_i)$.
(i.e., the “**bound**” step)

BRANCH AND BOUND

- Then we just repeat:
- For each iteration of the main loop,
 - Pop a state from the priority queue.
 - Partition the set into subsets (**branch**).
 - Push a new state for each subset with priority given by $g(\cdot)$ (**bound**).
- Eventually, we will pop a state with a set containing a single element $\{x\}$.
- We can compute $f(x)$ and check if it's larger than the priority of the next state in the queue.
- If so, then x is necessarily the optimal object in S .
 - Since the priority of the next state in the queue is an upper bound on $f(y)$ for all other objects y .

BRANCH AND BOUND

- Then we just repeat:
- For each iteration of the main loop,
 - Pop a state from the priority queue.
 - Partition the set into subsets (**branch**).
 - Push a new state for each subset with priority given by $g(\cdot)$ (**bound**).
- Eventually, we will pop a state with a state containing a single element $\{x\}$.
- The first such x may not be strictly more optimal than all other objects y .
 - There may be other objects y such that $f(x) = f(y)$.
- We can continue the branch-and-bound main loop to find the top- k objects.

CKY PARSING AS BRANCH AND BOUND?

- How can we formulate CKY as a branch-and-bound algorithm?
- S is the set of all syntax trees (both **valid** and **invalid**) for a given sentence.
(an invalid syntax tree would be one containing a rule not in the grammar)
- Recall each state in CKY is a span (i, j) .
 - This state represents the set of all syntax trees for the given sentence that contains a valid subtree for the subsequence starting at **i** and ending at **j** .
- Eventually, we reach the span $(0, n)$ which represents the set of all valid syntax trees for the full sentence.
- What is the “**branch**” step?
 - When processing the state (i, j) , we add a new state to the queue for each (i, m) for **$m > j$** and for each (m, j) for **$m < i$** .

CKY PARSING AS BRANCH AND BOUND?

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- Recall each state in CKY is a span (i, j) .
 - This state represents the set of all syntax trees for the given sentence that contains a valid subtree for the subsequence starting at i and ending at j .
- What is the “**bound**” $g(x)$?
 - We can set it to $g(x) = i - j$ so that shorter spans have higher priority.
 - But we must make sure $g(x)$ is an upper bound of the objective function.
(the objective function is 1 iff x is a valid parse of the whole sentence)
 - So we can simply set $g(x) = i - j + n + 1$.

EARLEY PARSING AS BRANCH AND BOUND?

- How can we formulate Earley parsing as a branch-and-bound algorithm?
- Recall each state in Earley contains a rule $A \rightarrow B_1 \dots B_m \cdot B_{m+1} \dots B_n$ with start position i and current position k .
 - This state represents the set of all syntax trees for the given sentence that contains a subtree for the subsequence starting at i ,
 - Where the subtree has root A ,
 - And this subtree has valid subtrees with roots $B_1 \dots B_m$ up to position k ,
 - And subtrees (valid or invalid) subtrees with roots $B_{m+1} \dots B_n$ after position k .

EARLEY PARSING AS BRANCH AND BOUND?

- How can we formulate Earley parsing as a branch-and-bound algorithm?
- What is the “branch” step?
 - Depending on the current state, we either do prediction, scanning, or completion.
- What is the “bound”?
 - As stated earlier, we can be flexible about the order we visit states.
 - We can use a heuristic and frame the problem as an A* search.
 - E.g., Lee et al. (2016) train a neural network to predict the bound.
 - Resulting in a faster parser (with fewer iterations).
 - In general, tighter bounds leads to faster searching.
 - I.e, $g(S)$ is closer to $\max_{x \in S} f(x)$

STRUCTURED PREDICTION

- **Structured prediction** is the task where the output is structured.
 - E.g., syntax trees, sequences, graphs, tables, etc.
- This task usually involves discrete optimization/search,
 - For example via algorithms like branch-and-bound.
- **Sequence prediction** is a kind of structured prediction.
 - Let's say we have some objective function f over sequences of items.
 - E.g., this could be a sequence of words.
 - E.g., Traveling Salesman Problem: suppose we need to make n deliveries in n different cities. In what order should we visit the cities to minimize the overall distance traveled?
- Goal: Find the sequence x_1, \dots, x_n such that $f(x_1, \dots, x_n)$ is maximized.
 - In traveling salesman, f is the negative distance.

SEQUENCE PREDICTION

- We can apply branch-and-bound to autoregressive sequence prediction.
- The set S is the set of all sequences of length n .
- Each state is a partial sequence: x_1, \dots, x_k
 - Represents the set of all sequences of length n that start with x_1, \dots, x_k .
 - E.g., the first k cities that we visit to make deliveries.
- What is the “branch” step?
 - For each possible next symbol x_{k+1} , we create a new state x_1, \dots, x_{k+1} .
- What is the “bound”?
 - The simplest bound is the total cost so far:

$$\begin{aligned} -\text{distance}(x_1, \dots, x_n) &\leq -\text{distance}(x_1, \dots, x_k), \\ &= -\text{distance}(x_1, x_2) - \text{distance}(x_2, x_3) - \dots - \text{distance}(x_{k-1}, x_k). \end{aligned} \quad 87$$

SEQUENCE PREDICTION

- This algorithm is too **slow**.
- In the worst case, we need a number of branches exponential in n .
 - We would need to search over all possible sequences of cities.
 - How many such sequences are there?
 - $n(n-1)(n-2)\dots(2)(1) = n!$
 - (there are n possible values for the first city, then there are $n-1$ possible values for the second city, etc...)

SEQUENCE PREDICTION

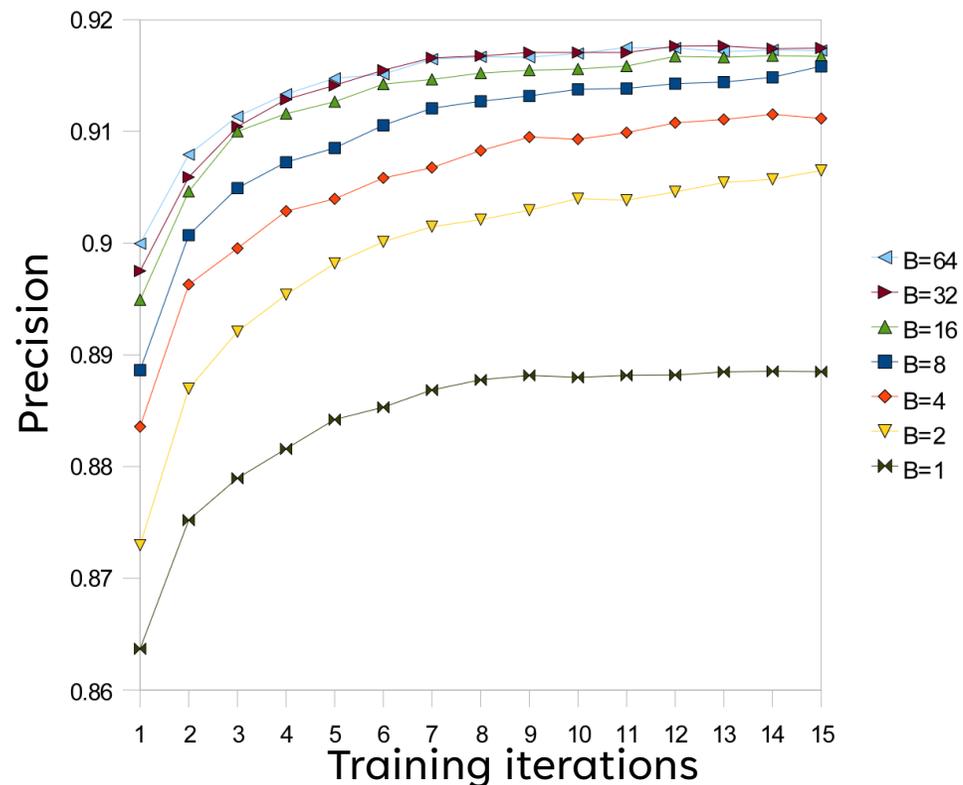
- How to make sequence prediction faster?
- We can trade optimality for performance.
 - We can limit the capacity of the priority queue.
 - Let B be the capacity.
 - After adding new states to the priority queue, simply remove the lowest priority states until only B elements remain.
- This is called **beam search**,
 - And B is the **beam width** or **beam size**.
- If $B = 1$, we have **greedy search**.
- If $B = \infty$, we recover exact search.

BEAM SEARCH IN PARSING

- Beam search can also be used in parsing.
- Used when there is an objective function over syntax trees.
 - E.g., a model that assigns probabilities to syntax trees.
 - This would be very useful in choosing among ambiguous parses.
 - E.g., in ‘Sally caught a butterfly with a net,’ who has the net?
- Can significantly increase parsing speed if there are many ambiguous parses.

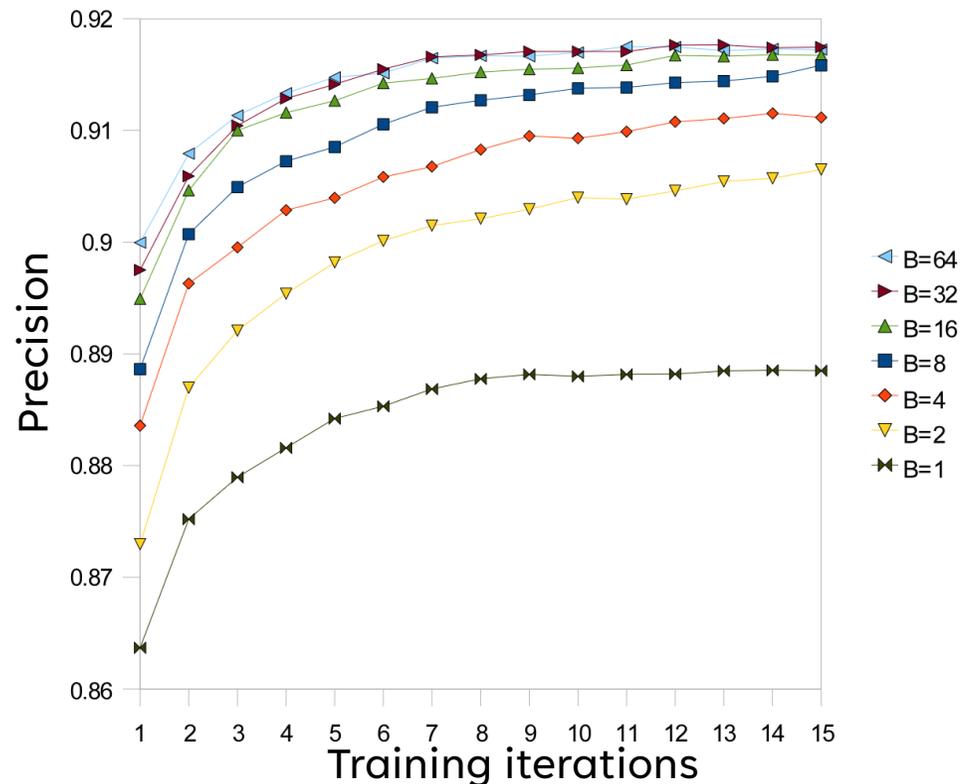
BEAM SEARCH IN PARSING

- Zhang and Clark (2008) used beam search for parsing.
- They used a neural model to assign probabilities to parser outputs.



BEAM SEARCH IN PARSING

- They measured precision vs number of training iterations for the neural model vs beam size.



The top-left portion of the slide features a series of thin, light-brown lines that intersect to form several overlapping, irregular polygons. These lines are scattered across the upper-left quadrant, creating a complex, abstract geometric pattern.

QUESTIONS?