

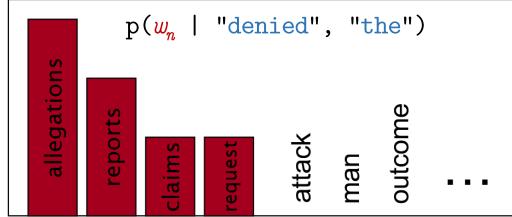
Lecture 4: Recurrent Neural Networks

PREVIOUSLY: DATA SPARSITY AND OVERFITTING

How do we resolve the data sparsity issue with n-gram models?

 E.g., we have a 3-gram model where we have seen the following phrases in the training data:

- "denied the allegations" 3 times
- "denied the reports" 2 times
- "denied the claims" 1 time
- "denied the request" 1 time
- No other instances of "denied the ____"

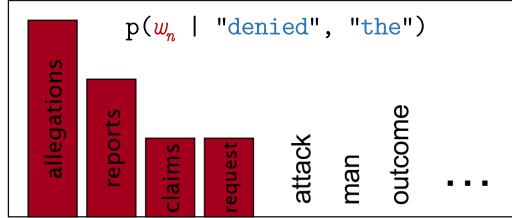


PREVIOUSLY: DATA SPARSITY AND OVERFITTING

- How do we resolve the data sparsity issue with n-gram models?
- One idea is called smoothing:
- The intuition is to "smooth" out the distribution of the next word, so that no word has probability 0.

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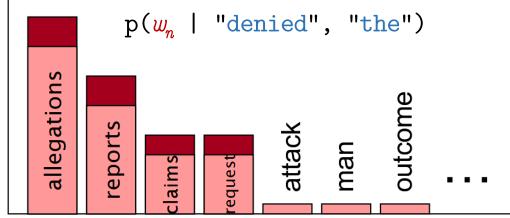


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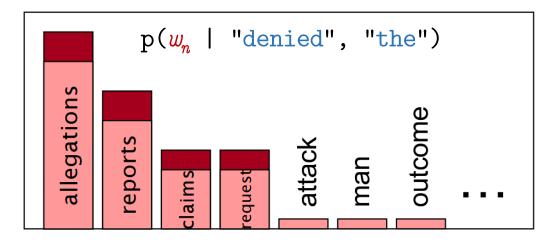
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SMOOTHING

- How do we resolve the data sparsity issue with n-gram models?
- Laplace smoothing (also called add-one smoothing):

$$p(w_n \mid w_1, ..., w_{n-1}) = \frac{(\# \text{ of times } w_n \text{ appeared after } w_1, ..., w_{n-1}) + 1}{(\# \text{ of times } w_1, ..., w_{n-1} \text{ appeared}) + V}$$



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```
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```

- This is simple, but doesn't work well in language modeling.
 - Consider the 4-gram model trained on Shakespeare.
 - For almost all 4-grams in the test set, the numerator in the above expression is 1.
- It is useful in other tasks, however.

BACKOFF

- How do we resolve the data sparsity issue with n-gram models?
- Another idea: Simultaneously use multiple n-gram models, with smaller n.

$$p(\textbf{\textit{w}}_n \mid \textbf{\textit{w}}_1, ..., \textbf{\textit{w}}_{n-1}) = \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared after } \textbf{\textit{w}}_1, ..., \textbf{\textit{w}}_{n-1}}{\text{\# of times } \textbf{\textit{w}}_n, \text{ appeared}} \text{ if } \textbf{\textit{w}}_1, ..., \textbf{\textit{w}}_n \text{ occurs in data}$$

$$= \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared after } \textbf{\textit{w}}_2, ..., \textbf{\textit{w}}_{n-1}}{\text{\# of times } \textbf{\textit{w}}_n, \text{ appeared after } \textbf{\textit{w}}_{n-1}} \text{ if } \textbf{\textit{w}}_2, ..., \textbf{\textit{w}}_n \text{ occurs in data}$$

$$= \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared after } \textbf{\textit{w}}_{n-1}}{\text{\# of times } \textbf{\textit{w}}_{n-1} \text{ appeared}} \text{ if } \textbf{\textit{w}}_{n-1}, \textbf{\textit{w}}_n \text{ occurs in data}$$

$$= \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared}}{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared}} \text{ otherwise.}$$

INTERPOLATION

- How do we resolve the data sparsity issue with n-gram models?
- Another idea: Use multiple n-gram models, with interpolation.

```
p(\textbf{\textit{w}}_n \mid \textbf{\textit{w}}_1, ..., \textbf{\textit{w}}_{n-1}) = \lambda_n \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared after } \textbf{\textit{w}}_1, ..., \textbf{\textit{w}}_{n-1}}{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared after } \textbf{\textit{w}}_2, ..., \textbf{\textit{w}}_{n-1}} \\ + \lambda_{n-1} \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared after } \textbf{\textit{w}}_2, ..., \textbf{\textit{w}}_{n-1}}{\text{\# of times } \textbf{\textit{w}}_2, ..., \textbf{\textit{w}}_{n-1} \text{ appeared}} \\ \dots \\ + \lambda_2 \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appeared after } \textbf{\textit{w}}_{n-1}}{\text{\# of times } \textbf{\textit{w}}_{n-1} \text{ appeared}} \\ + \lambda_1 \frac{\text{\# of times } \textbf{\textit{w}}_n \text{ appears}}{\text{total number of words}}
```

INTERPOLATION

- This type of model is called a mixture model.
- Equivalent to first rolling an n-sided die to choose which n-gram to sample from, and then sampling from the corresponding n-gram model.

```
p(w_n \mid w_1, ..., w_{n-1}) = p(w_n \mid w_1, ..., w_{n-1}, \text{ choose 1-gram}) \text{ p(choose 1-gram)} + ... + p(w_n \mid w_1, ..., w_{n-1}, \text{ choose n-gram)} \text{ p(choose n-gram)},
```

By the law of total probability.

```
= p(w_n \mid w_1, ..., w_{n-1}, \text{ choose 1-gram}) \lambda_1
+ ... + p(w_n \mid w_1, ..., w_{n-1}, \text{ choose n-gram}) \lambda_n.
```

DATA SPARSITY AND OVERFITTING

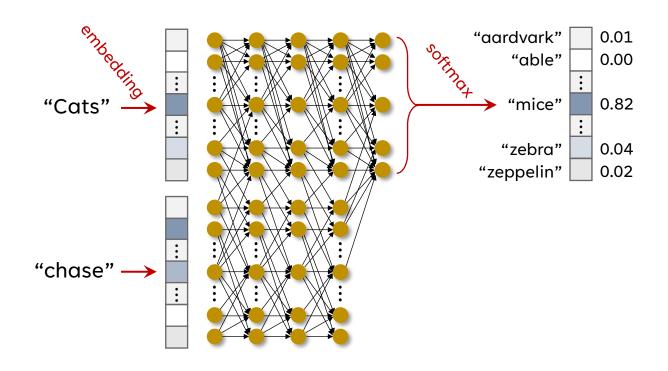
(some NLP history)

- Backoff performs better when combined with smoothing.
 - Kneser-Ney smoothing
 - Interpolated Kneser-Ney
 - Skip n-grams
- Another idea to address the data sparsity issue, is to use a different machine learning model.
 - Perhaps a neural network?
- Smoothing/interpolation superseded by neural language models.

BETTER METHODS FOR TEXT CLASSIFICATION

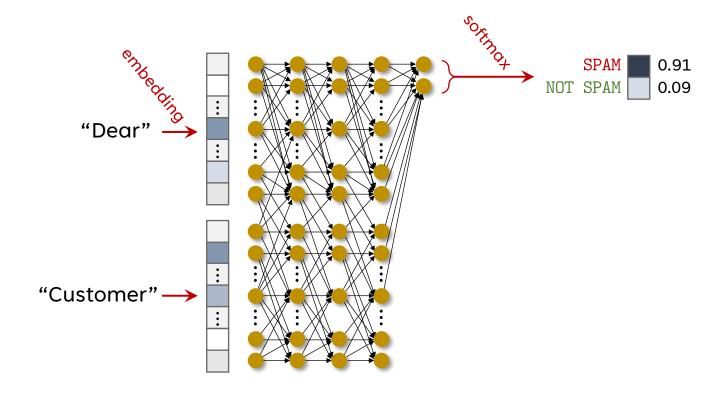
- We previously discussed language modeling:
 - The task of predicting the next word, given previous words.
 - n-gram models are simple but not very accurate for small n.
 - They suffer from data sparsity and overfitting for large n.
- Are there alternative machine learning models that would be better?
- Maybe MLP?

MLP (?) FOR LANGUAGE MODELING



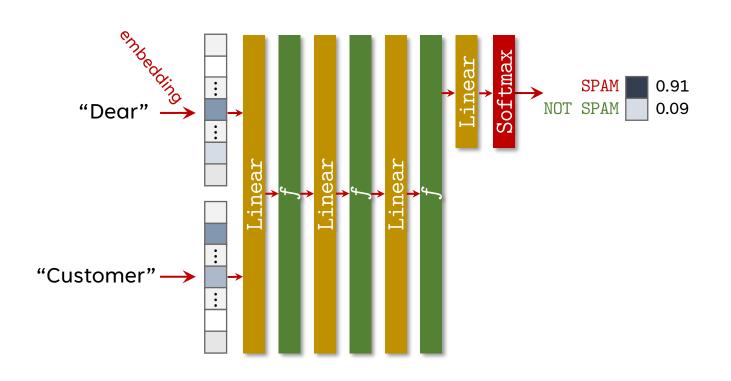
- Embed each input word into a real-valued vector with dimension *d*.
- Concatenate the embedding vectors and input into MLP.
- Input layer has dimension $N \cdot d$.
- Output layer has dimension *V*.
- Here, the MLP has 3 hidden layers.

MLP (?) FOR SPAM DETECTION



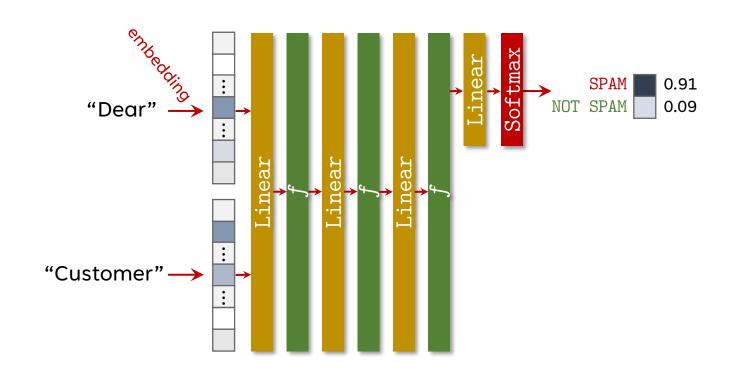
 MLPs can be used for other text classification tasks.

MLP (?) FOR SPAM DETECTION



- MLPs can be used for other text classification tasks.
- Each linear layer computes the function: Linear(x) = Wx + b
- Each nonlinearity (f) computes an activation function element-wise.
- Example activation functions:
 - Sigmoid
 - tanh
 - ReLU

MLP (?) FOR SPAM DETECTION



- MLPs can be used for other text classification tasks.
- Potential disadvantages?
 - Thoughts?
 - How many parameters (weights) are there?
 - $(Nd)^2L + NdD_{output}$ L is the number of hidden layers.
 - *N* is the maximum number of input words.

MLPS (?) FOR TEXT CLASSIFICATION

- More expressive machine learning models are more prone to overfitting.
 - They need more data to train.
 - I.e., they are less data efficient.
- Number of parameters/weights is a measure of model expressiveness.
- Pure MLPs are not very data efficient.
 - Especially if the embedding dimension d or input length N is very large.
 - E.g., in GPT-3, d = 12288, N = 4096.

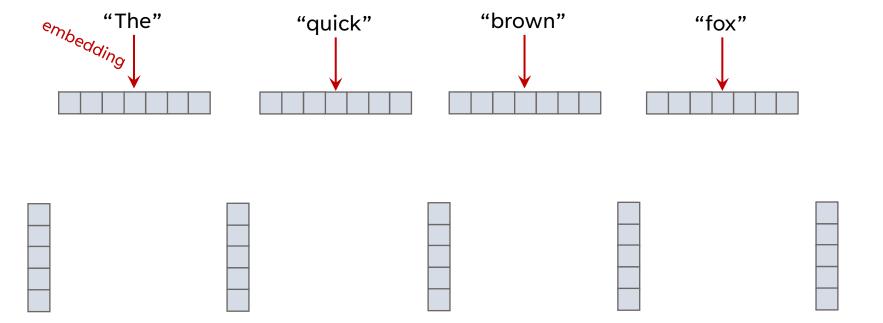
ALTERNATIVE NEURAL ARCHITECTURES

- But there is a very large space of different neural architectures.
- One natural proposal is to model the sequential nature of language.
 - Humans understand language word-by-word.
 - Humans hear/read each word and update an internal representation in their brain.
- Can we capture this kind of processing in a neural architecture?

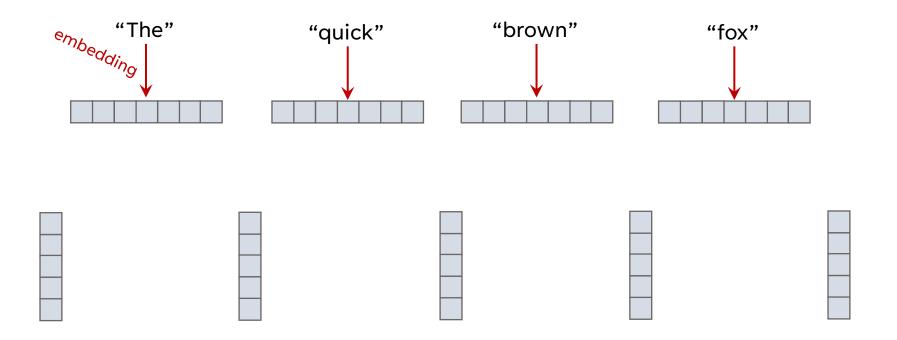
• Recurrent neural networks (RNNs; Elman 1990) attempt to capture this sequential (word-by-word) processing.

"The" "quick" "brown" "fox"

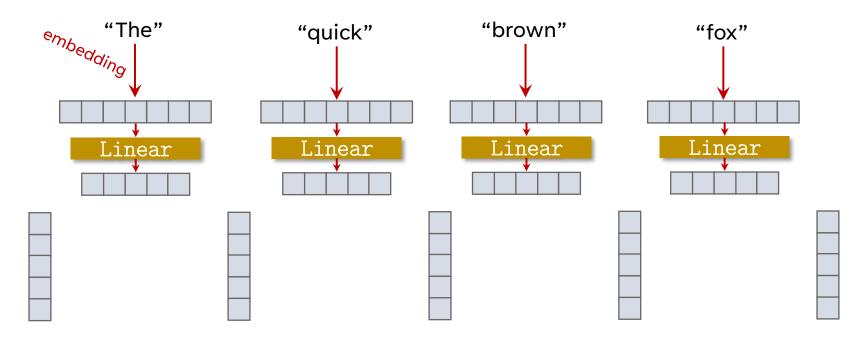
- Embed each input word into vectors of dimension d_{emb} .
- The RNN keeps a hidden state vector with dimension d_{hid} .



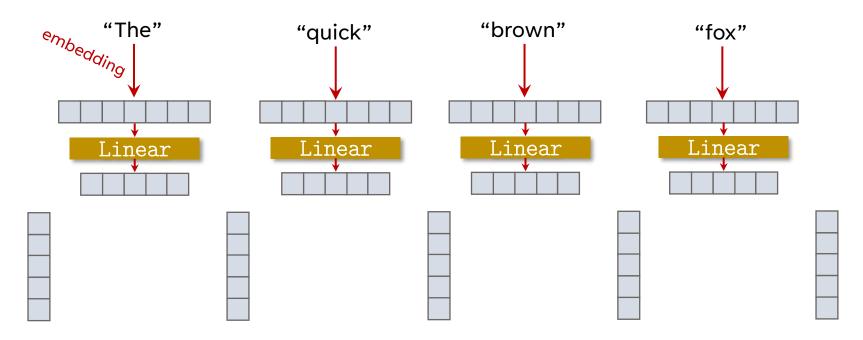
• The RNN combines each word with the previous hidden state, to produce the next hidden state.



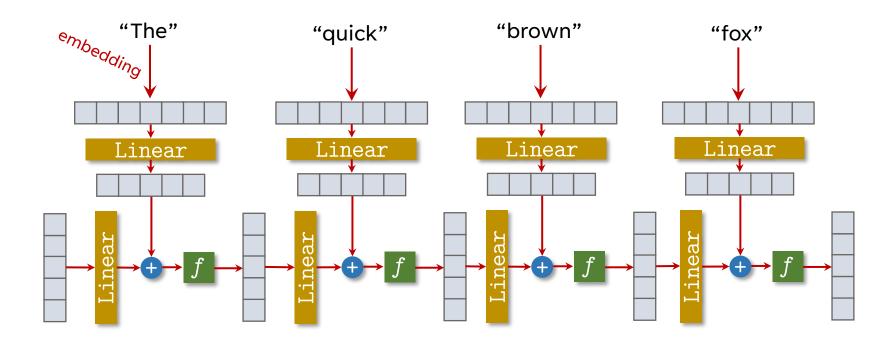
- To do so, we need to convert the embeddings into d_{hid} -dimensional vectors.
- We do this with a linear layer.



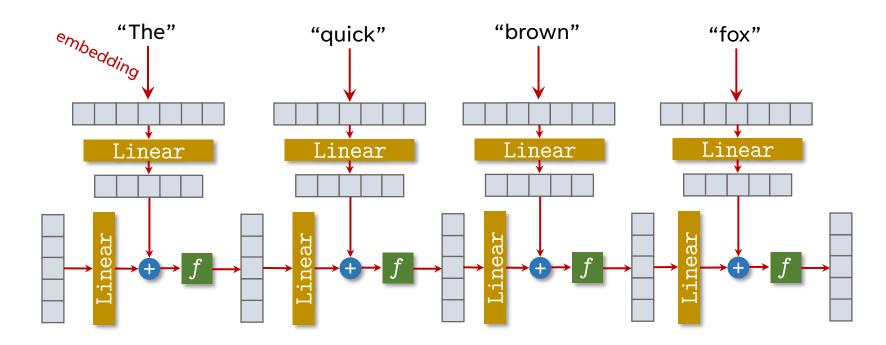
- Importantly, these linear layers are coupled.
- Each linear layer has the same weights as the other linear layers.



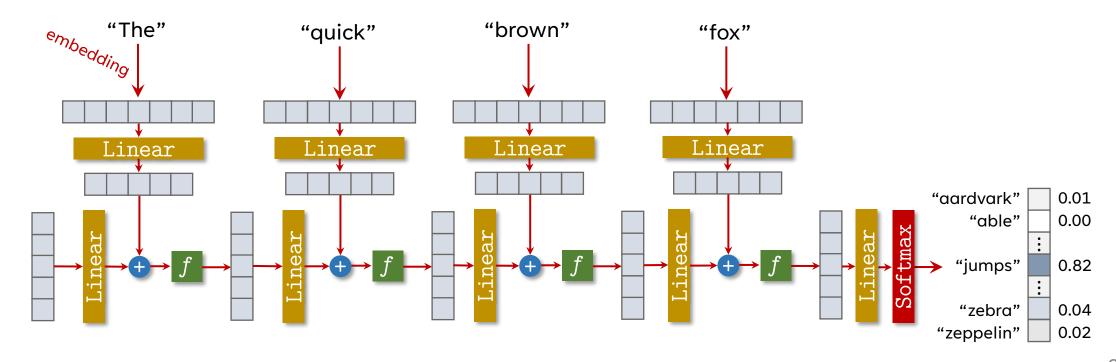
• Now the word vectors and hidden state have the same dimension, we combine them to produce the next hidden state.



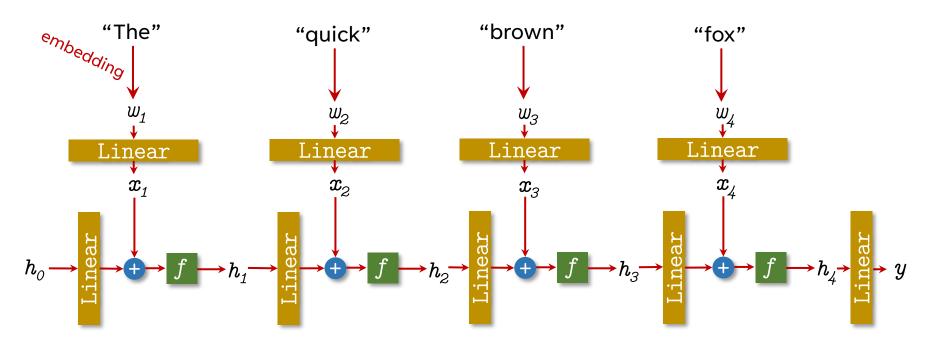
• The linear layers acting on the hidden states are also coupled.



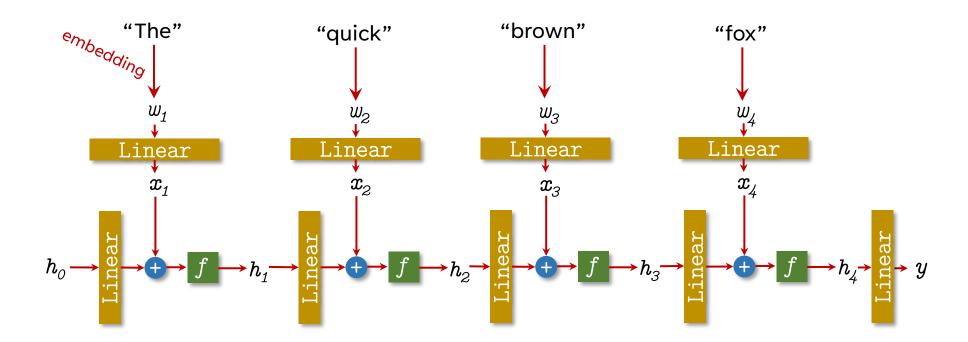
- Once we have the last hidden state, we can use it to make a prediction.
- In the example, we have a language modeling/next-word prediction task.



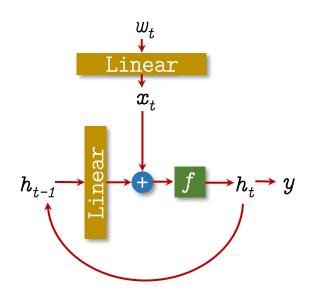
- It's often easier to depict neural architectures symbolically.
- Note h_0 is often set to a vector of zeros, but it can also be learned.



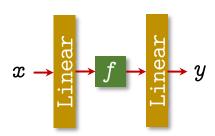
• Why "recurrent"?



- Why "recurrent"?
- Converting from this into the feedforward (i.e., directed acyclic) form is called "unfolding in time".



- We train RNNs the same way we train most neural networks:
- Gradient descent, using backprop to compute gradients.
- How do we compute gradients when some parameters are coupled?
- Consider the following simple MLP: (no coupled parameters)



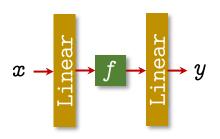
$$y = b_2 + W_2 \cdot f(b_1 + W_1 x)$$

- Suppose we have a training example (\hat{x}, \hat{y}) .
- And we have some loss function $L(y, \hat{y})$.
- We can compute the gradient of the loss:
- Similarly, compute gradients for b_1 and b_2 .

$$\nabla_{W_2} L(y, \hat{y}) = L'(y, \hat{y}) \cdot \nabla_{W_2} y$$

$$= L'(y, \hat{y}) \cdot \nabla_{W_2} (b_2 + W_2 \cdot f(b_1 + W_1 \hat{x}))$$

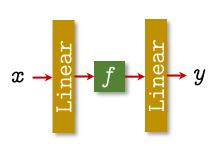
$$= L'(y, \hat{y}) \cdot f(b_1 + W_1 \hat{x})$$



$$y = b_2 + W_2 \cdot f(b_1 + W_1 x)$$

$$\begin{split} \nabla_{\mathcal{W}_{1}} L(y, \hat{y}) &= L'(y, \hat{y}) \cdot \nabla_{\mathcal{W}_{1}} y \\ &= L'(y, \hat{y}) \cdot \nabla_{\mathcal{W}_{1}} (b_{2} + \mathcal{W}_{2} \cdot f(b_{1} + \mathcal{W}_{1} \hat{x})) \\ &= L'(y, \hat{y}) \cdot \mathcal{W}_{2} \cdot \nabla_{\mathcal{W}_{1}} f(b_{1} + \mathcal{W}_{1} \hat{x}) \\ &= L'(y, \hat{y}) \cdot \mathcal{W}_{2} \cdot f'(b_{1} + \mathcal{W}_{1} \hat{x}) \cdot \nabla_{\mathcal{W}_{1}} (b_{1} + \mathcal{W}_{1} \hat{x})^{\mathsf{T}} \\ &= L'(y, \hat{y}) \cdot \mathcal{W}_{2} \cdot f'(b_{1} + \mathcal{W}_{1} \hat{x}) \cdot \hat{x}^{\mathsf{T}} \end{split}$$

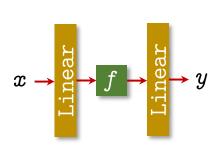
- But now let's consider the case where the two linear layers are coupled.
- Notice the result is just the sum of the gradients from the uncoupled case.
- Gradient accumulation



$$y = b_1 + W_1 \cdot f(b_1 + W_1 x)$$

$$\begin{split} \nabla_{W_{1}} L(y, \hat{y}) &= L'(y, \hat{y}) \cdot \nabla_{W_{1}} y \\ &= L'(y, \hat{y}) \cdot \nabla_{W_{1}} (b_{1} + W_{1} \cdot f(b_{1} + W_{1} \hat{x})) \\ &= L'(y, \hat{y}) \cdot (W_{1} \cdot f'(b_{1} + W_{1} \hat{x}) \cdot \hat{x}^{T} + f(b_{1} + W_{1} \hat{x})) \\ &= L'(y, \hat{y}) \cdot W_{1} \cdot f'(b_{1} + W_{1} \hat{x}) \cdot \hat{x}^{T} + L'(y, \hat{y}) \cdot f(b_{1} + W_{1} \hat{x}) \end{split}$$

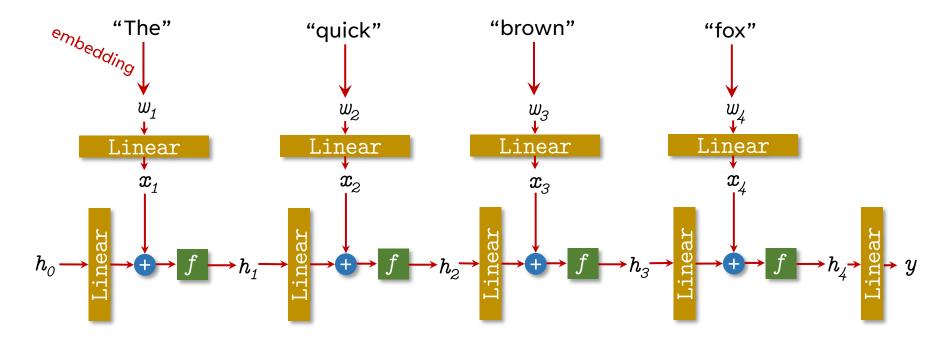
- Note that most modern ML libraries will compute gradients automatically.
- But it's good to know what's happening under the hood.
 - Useful if something goes wrong -> debugging.
 - Also useful to think about new techniques for better ML.



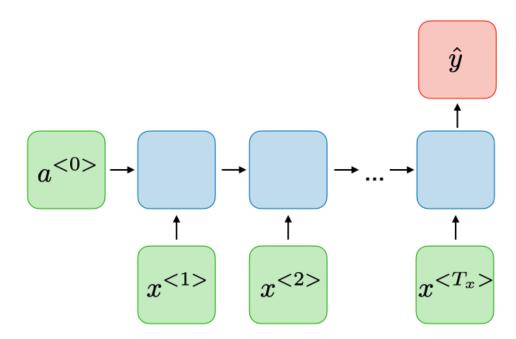
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$$\begin{split} \nabla_{W_{1}}L(y,\hat{y}) &= L'(y,\hat{y}) \cdot \nabla_{W_{1}}y \\ &= L'(y,\hat{y}) \cdot \nabla_{W_{1}}(b_{1} + W_{1} \cdot f(b_{1} + W_{1}\hat{x})) \\ &= L'(y,\hat{y}) \cdot (W_{1} \cdot f'(b_{1} + W_{1}\hat{x}) \cdot \hat{x}^{T} + f(b_{1} + W_{1}\hat{x})) \\ &= L'(y,\hat{y}) \cdot W_{1} \cdot f'(b_{1} + W_{1}\hat{x}) \cdot \hat{x}^{T} + L'(y,\hat{y}) \cdot f(b_{1} + W_{1}\hat{x}) \end{split}$$

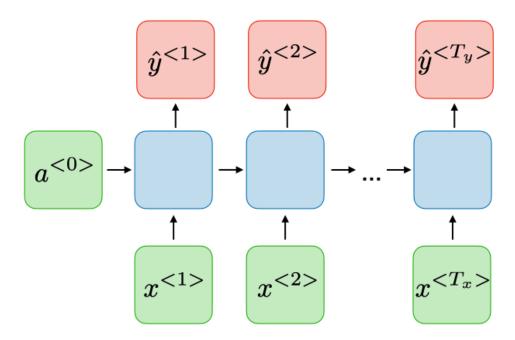
- RNNs have a very wide variety of applications.
- Beyond simple text classification.



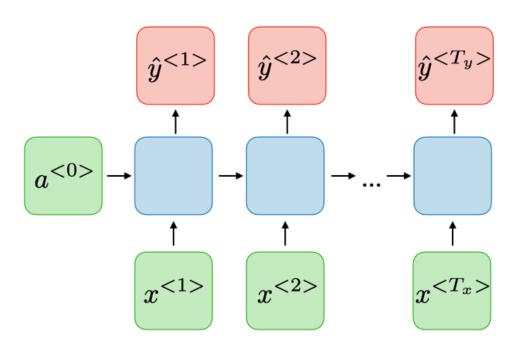
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- We can make more than one prediction.
- For example, we can make a prediction per input word.



- Example tasks:
 - Part-of-speech tagging, named-entity recognition



Input: The quick brown fox jumped.

Output: DET ADJ ADJ NN V

When training, for each example, we sum the loss over all predictions.

Example prediction:

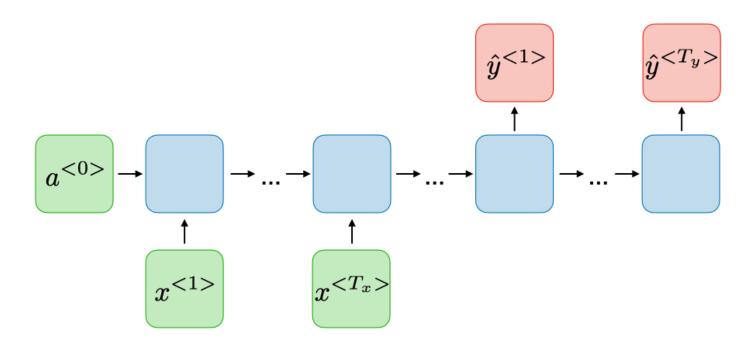
Input: The quick brown fox jumped.

Prediction: DET ADJ NN NN V

 $Total\ loss = L(DET, DET) + L(ADJ, ADJ) + L(ADJ, NN) + L(NN, NN) + L(V, V)$

RNN APPLICATIONS

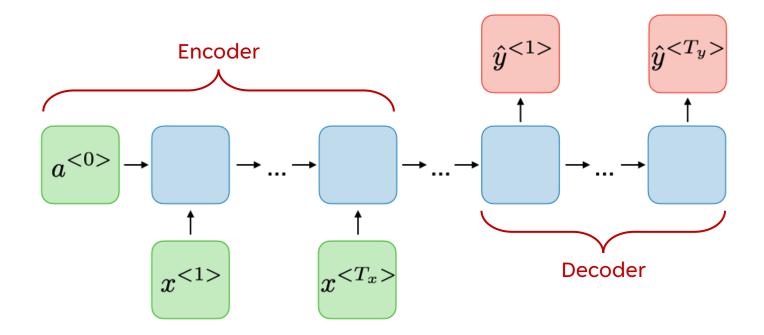
• The number of output predictions doesn't need to match the number of input predictions.



RNN APPLICATIONS

- Example tasks:
 - Machine translation

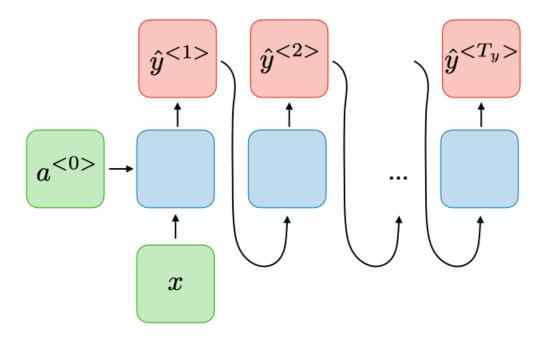
Input: The quick brown fox jumped over the lazy dog.
Output: 素早い茶色のキツネは怠け者の犬を飛び越えました。



RNN APPLICATIONS

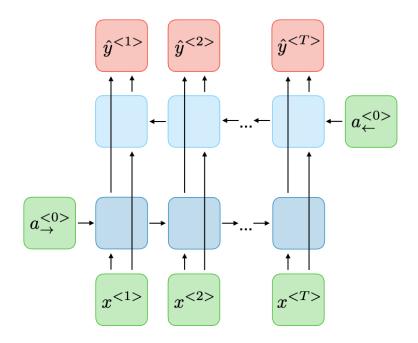
- It can be used in non-text applications.
- Example task: music generation





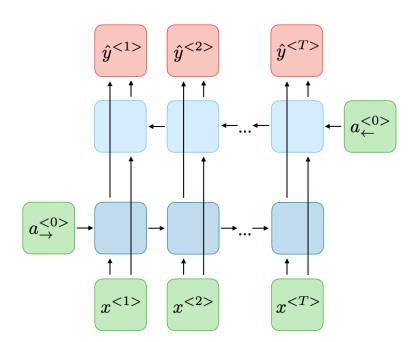
BIDIRECTIONAL RNN

• Bidirectional RNNs (BiRNNs) can be used in tasks where we want to gather information from words on both the left and right sides.



BIDIRECTIONAL RNN

This is useful in the masked language modeling task.
 (a good unidirectional RNN could also solve this task)



Input: The quick brown ___ jumped.

Output: fox

Input: I am ___.

Output: running

Input: I am ___ hungry.

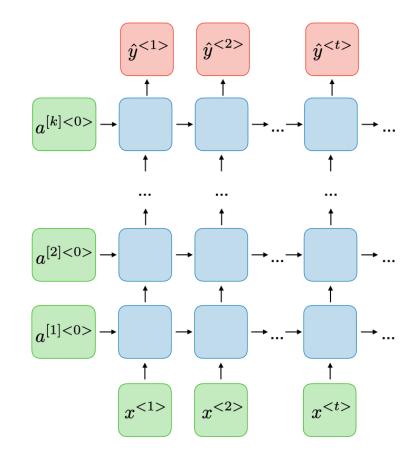
Output: so

Input: I am ___ hungry; I just ate.

Output: not

DEEP RNN

• We can stack many layers of RNNs.



RNN GRADIENTS

Suppose we have a long RNN (lots of tokens).

$$y = g_n(g_{n-1}(\ldots g_2(g_1(x_1, W_1))\ldots))$$

- Each g_i is an RNN "unit", written more simply.
- x_1 is the first word, and W_1 is the weight matrix in the linear layer after x_1 .
- What is the gradient with respect to W_1 ?

$$\nabla_{W_{1}} \mathbf{y} = g_{n}'(...) \cdot \nabla_{W_{1}} g_{n-1}(...)$$

$$= g_{n}'(...) \cdot g_{n-1}'(...) \cdot \nabla_{W_{1}} g_{n-2}(...)$$

$$= ... = g_{n}'(...) \cdot g_{n-1}'(...) \cdot ... \cdot g_{2}'(...) \cdot g_{1}'(...) \cdot \mathbf{x}_{1}^{\mathrm{T}}$$

RNN GRADIENTS

- Note that this is a product containing many terms.
- If the terms are > 1, their product will grow exponentially in n.
- If the terms are < 1, their product will shrink to 0 exponentially in n.
- This is called the exploding or vanishing gradient problem.
- This is also an issue for very deep networks (containing many layers).

$$\nabla_{W_{1}} \mathbf{y} = g_{n}'(...) \cdot \nabla_{W_{1}} g_{n-1}(...)$$

$$= g_{n}'(...) \cdot g_{n-1}'(...) \cdot \nabla_{W_{1}} g_{n-2}(...)$$

$$= ... = g_{n}'(...) \cdot g_{n-1}'(...) \cdot ... \cdot g_{2}'(...) \cdot g_{1}'(...) \cdot \mathbf{x}_{1}^{\mathrm{T}}$$

VANISHING/EXPLODING GRADIENTS

- How do we solve this problem?
- Pick activation functions whose derivatives are 1 (ReLU).
- Gradient clipping:

If the gradient vector v has magnitude larger than v_{max} , divide it by $||v||/v_{max}$, so that its magnitude is at most v_{max} .

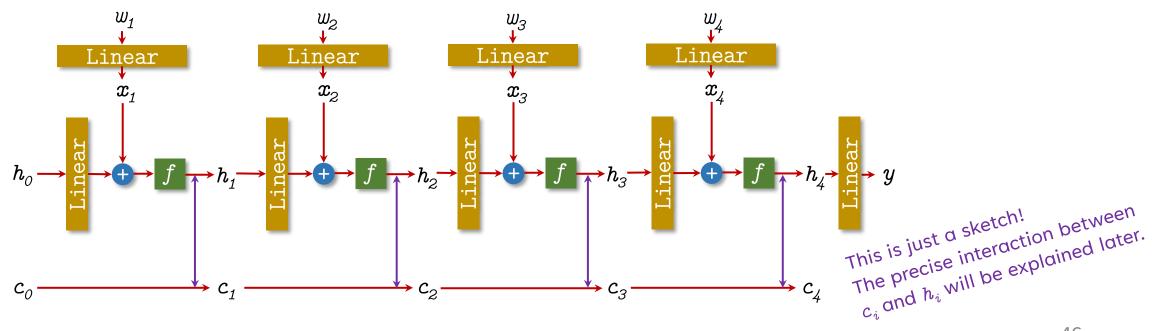
$$\nabla_{W_{1}} \mathbf{y} = g_{n}'(...) \cdot \nabla_{W_{1}} g_{n-1}(...)$$

$$= g_{n}'(...) \cdot g_{n-1}'(...) \cdot \nabla_{W_{1}} g_{n-2}(...)$$

$$= ... = g_{n}'(...) \cdot g_{n-1}'(...) \cdot ... \cdot g_{2}'(...) \cdot g_{1}'(...) \cdot \mathbf{x}_{1}^{\mathsf{T}}$$

VANISHING/EXPLODING GRADIENTS

- Another solution is to change the architecture.
- Long short-term memory (LSTM; Hochreiter and Schmidhuber 1997)



LONG SHORT-TERM MEMORY

- Key idea is that the updates to the c_i stream are additive.
- So gradients of c_i do not get very large or very small with increasing \emph{n} .
- We will go into further detail next lecture.

